HW#12 SOLUTIONS.

$\boxed{\S 9.2}$

(18) $\begin{cases} x = 40t + 5 \\ y = 20 + 3t - 16t^2 \end{cases}$

Sol) $x'(t) = 40$

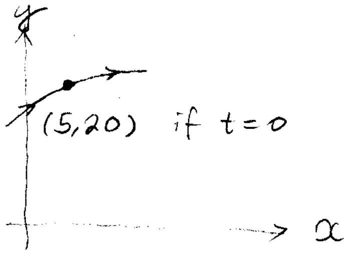
$y'(t) = 3 - 32t$

(a) $t = 0$

$\vec{V}(0) = (x'(0), y'(0)) = (40, 3)$

$S(0) = \sqrt{1609}$

right / up



(b) $t = 2$.

$\vec{V}(2) = (40, -61)$

$S(0) = \sqrt{1600 + 3721} = \sqrt{5321}$

right / down

(26) $\begin{cases} x = t\sin t \\ y = t\cos t, \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}. \end{cases}$

Sol) $\pm A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} y(t)\, x'(t)\, dt$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} t\cos t\,(\sin t + t\cos t)\, dt$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (t\cos t\sin t + t^2\cos^2 t)\, dt$

even functions.

$\left.\begin{array}{l} \sin(2t) = 2\sin t\cos t \\ \cos^2(t) = \dfrac{1+\cos(2t)}{2} \end{array}\right\}$ so

$2\int_0^{\frac{\pi}{2}} \left[t\cdot\frac{1}{2}\sin(2t) + t^2\,\frac{1+\cos(2t)}{2}\right] dt$

$= \underbrace{\int_0^{\frac{\pi}{2}} t\sin(2t)\,dt}_{a} + \underbrace{\int_0^{\frac{\pi}{2}} t^2\,dt}_{b} + \underbrace{\int_0^{\frac{\pi}{2}} t^2\cos(2t)\,dt}_{c}$

(b) $\left[\frac{t^3}{3}\right]_0^{\frac{\pi}{2}} = \frac{1}{3}\cdot\frac{\pi^3}{8} = \frac{\pi^3}{24}$

(a) $u = t \qquad dv = \sin(2t)\,dt$

$du = dt \qquad v = -\frac{1}{2}\cos(2t)$

$\left[-\frac{t}{2}\cos(2t)\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{1}{2}\cos(2t)\,dt$

$= -\frac{\pi}{4}(-1) + \left[\frac{1}{4}\sin(2t)\right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$

(c) $u = t^2 \qquad dv = \cos(2t)\,dt$

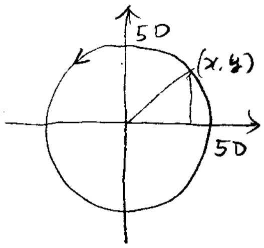
$du = 2t\,dt \qquad v = \frac{1}{2}\sin(2t)$

$\left[\frac{t^2}{2}\sin(2t)\right]_0^{\frac{\pi}{2}} - \underbrace{\int_0^{\frac{\pi}{2}} t\sin(2t)\,dt}_{a}$

$= 0 - \frac{\pi}{4} = -\frac{\pi}{4}$

$\therefore$ (a) + (b) + (c) $= \frac{\pi^3}{24} > 0$

36 Sol)



round once in 3 minutes.

$$\begin{cases} x(t) = 50\cos\left(\frac{2\pi}{3}t\right) \\ y(t) = 50\sin\left(\frac{2\pi}{3}t\right) \end{cases} \quad 0 \le t \le 3$$

$$x'(t) = -\frac{100\pi}{3}\sin\left(\frac{2\pi}{3}t\right)$$

$$y'(t) = \frac{100\pi}{3}\cos\left(\frac{2\pi}{3}t\right)$$

$$\text{speed} = \frac{100\pi}{3} \ (f/m)$$

$$\approx 104.67 \ (f/m)$$

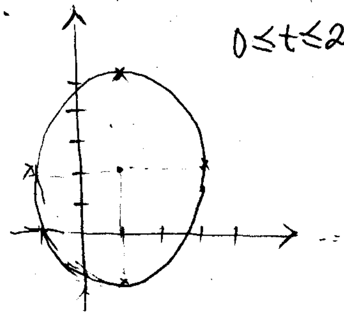
§9.3

2. $\begin{cases} x = 1 - 2\cos t \\ y = 2 + 3\sin t \end{cases}$

Sol) $\cos(t) = \frac{x-1}{2}$, $\sin(t) = \frac{y-2}{3}$

$$\left(\frac{x-1}{2}\right)^2 + \left(\frac{y-2}{3}\right)^2 = 1$$

an ellipse. round once if $0 \le t \le 2\pi$.



$x'(t) = 2\sin(t)$, $y'(t) = 3\cos(t)$.

$$\int_0^{2\pi} \sqrt{4\sin^2(t) + 9\cos^2(t)}\,dt$$

$$= \int_0^{2\pi} \sqrt{9 - 5\sin^2(t)}\,dt \ \underset{\text{CAS}}{\approx} \Rightarrow 3E\left(t \,\middle|\, \tfrac{5}{9}\right)$$

elliptic integral of the second kind.

14 $\begin{cases} x = \pi t \\ y = 2\sqrt[4]{t} \end{cases}$ (0,0) $t=0$ → $(\pi, 2)$ $t=1$

Sol) $\text{Time} = \int_0^1 k\sqrt{\frac{x'(u)^2 + y'(u)^2}{y(u)}}\,du$

$x'(t) = \pi$

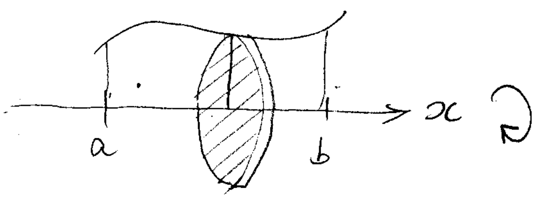
$y'(t) = 2 \cdot \frac{1}{4}t^{-\frac{3}{4}} = \frac{1}{2}t^{-\frac{3}{4}}$

$$\text{Time} = \int_0^1 k\sqrt{\frac{\pi^2 + \frac{1}{4}u^{-\frac{3}{2}}}{\frac{1}{2}u^{-\frac{3}{4}}}}\,du$$

$$= k\int_0^1 \sqrt{2\pi^2 u^{\frac{3}{4}} + \frac{1}{2}u^{-\frac{3}{4}}}\,du \approx \text{CAS}$$

22 $\begin{cases} x = t^2 - 1 \\ y = t^3 - 4t \end{cases}$, $0 \le t \le 2$, about x-axis.

Sol)



$dA = 2\pi|y|ds$,

where A = surface area, s = arc length

$$A = \int_0^2 2\pi\,|t^3 - 4t|\sqrt{(2t)^2 + (3t^2-4)^2}\,dt$$

$$(\because ds = \sqrt{x'(t)^2 + y'(t)^2}\,dt)$$

$$= 2\pi\int_0^2 (4t - t^3)\sqrt{9t^4 - 20t^2 + 16}\,dt \approx \text{CAS}$$