HW #4 Sol. 6.6

(2.) $\int_1^2 x^{-2/5}\,dx$

Sol) "Not" improper.

$\frac{1}{x^{2/5}} < \infty$ on $[1, 2]$.

(8.) $\int_0^1 x^{-4/3}\,dx$

Sol) $\frac{1}{x^{4/3}} \to \infty$ as $x \to 0^+$.

It's an improper integral.

So $\int_0^1 x^{-4/3}\,dx := \lim_{a\to 0^+} \int_a^1 x^{-4/3}\,dx$.

$\int_a^1 x^{-4/3}\,dx = \left[-3x^{-1/3}\right]_a^1$

$= -3\left(1 - a^{-1/3}\right) \to \infty$. as $a \to \infty$

(10.) $\int_1^\infty x^{-6/5}\,dx$.

Sol) $\int_1^\infty x^{-6/5}\,dx := \lim_{a\to\infty} \int_1^a x^{-6/5}\,dx$

$\int_1^a x^{-6/5}\,dx = \left[-5x^{-\frac{1}{5}}\right]_1^a$

$= -5\left(a^{-\frac{1}{5}} - 1\right) \to 5$ as $a \to \infty$

(12). $\int_1^5 \frac{2}{\sqrt{5-x}}\,dx$

Sol) $\sqrt{5-x} = 0$ at $x = 5$.

$\int_1^5 \frac{2}{\sqrt{5-x}}\,dx = \lim_{a\to 5^-} \int_1^a \frac{2}{\sqrt{5-x}}\,dx$,

$2\int_1^a (5-x)^{-\frac{1}{2}}\,dx = 2\left[2(5-x)^{\frac{1}{2}}\cdot(-1)\right]_1^a$

$= -4\left\{(5-a)^{\frac{1}{2}} - 2\right\} \to 8$ as $a \to 5$.

(14.) $\int_0^{\pi/2} \tan x\,dx$

Sol) $\tan\left(\frac{\pi}{2}\right) = \infty$.

$\int_0^{\pi/2} \tan x\,dx = \left[-\ln|\cos x|\right]_0^{\pi/2}$

$= \infty$. diverges.

(18) $\int_1^\infty x^2 e^{-2x}\,dx$

Sol) $u = x^2 \quad dv = e^{-2x}\,dx$

$du = 2x\,dx \quad v = -\frac{1}{2}e^{-2x}$

(18) "continued"

$$\int x^2 e^{-2x}\,dx = -\frac{1}{2}x^2e^{-2x} - \int -x e^{-2x}\,dx$$

$$\left(\begin{array}{ll} u = x & dv = e^{-2x}dx \\ du = dx & v = -\frac{1}{2}e^{-2x} \end{array}\right)$$

$$= -\frac{1}{2}x^2e^{-2x} + \left(-\frac{1}{2}\right)xe^{-2x} + \frac{1}{2}\int e^{-2x}dx$$

$$= -\frac{1}{2}x^2e^{-2x} - \frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$$

So

$$\left[-\frac{1}{2}x^2e^{-2x} - \frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right]_1^\infty$$

$$= \frac{1}{2}e^{-2} + \frac{1}{2}e^{-2} - \frac{1}{4}e^{-2} = \frac{3}{4}e^{-2}$$

(24) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2-1}\,dx$

Sol) $\displaystyle\int_0^1 \frac{1}{x^2-1}\,dx$

$$= \int_0^1 \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)dx$$

$$= \frac{1}{2}\Big[\ln|x-1| - \ln|x+1|\Big]_0^1 = -\infty$$

So $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2-1}\,dx$ diverges

(40) $\displaystyle\int_1^\infty \frac{x^2-2}{x^4+3}\,dx$

Sol) $\displaystyle\frac{x^2-2}{x^4+3} < \frac{x^2}{x^4} = \frac{1}{x^2}$

$\displaystyle\int_1^{\sqrt{2}} \frac{x^2-2}{x^4+3}\,dx$ converges

since $[1, \sqrt{2}]$ is finite.

$$\int_{\sqrt{2}}^\infty \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_{\sqrt{2}}^\infty$$

$$= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} < \infty.$$

So $\displaystyle\int_{\sqrt{2}}^\infty \frac{x^2-2}{x^4+3}\,dx$ converges.

Thus $\displaystyle\int_1^\infty \frac{x^2-2}{x^4+3}\,dx$ converges.