**Version A**

# Lecture 01 Practice Solutions

## Solutions

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1.

**Market research indicates that consumers will buy *x* thousand units of a particular kind of coffee maker when the unit price is**

$$p = -0.2x + 50$$

**dollars. What is the revenue function, *R*(*x*), for this production process?**

$R(X)$ = ____

Solution:

The demand function is $D(x) = -0.2x + 50$, so the revenue is

$$R(x) = xD(x) = -0.2x^2 + 50x$$

thousand dollars.

Correct answer is: $R(X) = -0.2x^2 + 50x$

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2.

**Market research indicates that consumers will buy *x* thousand units of a particular kind of coffee maker when the unit price is**

$$p = -0.15x + 52$$

**dollars. The cost of producing the *x* thousand units is**

$$C(x) = 3.04x^2 + 2.9x + 84$$

**thousand dollars. What is the profit function, *P*(*x*), for this production process?**

$P(x) =$ ____

Solution:

The demand function is $D(x) = -0.15x + 52$, so the revenue is

$$R(x) = xD(x) = -0.15x^2 + 52x$$

thousand dollars, and the profit is

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= -0.15x^2 + 52x - (3.04x^2 + 2.9x + 84) \\ &= -3.19x^2 + 49.1x - 84 \end{aligned}$$

thousand dollars.

Correct answer is: $P(x) = -3.19x^2 + 49.1x - 84$

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3.

**Market research indicates that consumers will buy $x$ thousand units of a particular kind of coffee maker when the unit price is**

$$p = -0.27x + 59$$

**dollars. The cost of producing the $x$ thousand units is**

$$C(x) = 2.33x^2 + 1.8x + 104$$

**thousand dollars. For what values of $x$ is production of the coffee makers profitable?**

Production is profitable for ____ .

Solution:

The demand function is $D(x) = -0.27x + 59$, so the revenue is

$$R(x) = xD(x) = -0.27x^2 + 59x$$

thousand dollars, and the profit is

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= -0.27x^2 + 59x - (2.33x^2 + 1.8x + 104) \end{aligned}$$

$= -2.6x^2 + 57.2x - 104$

thousand dollars.

Production is profitable when $P(x) > 0$. We find that

$$\begin{aligned} P(x) &= -2.6x^2 + 57.2x - 104 \\ &= -2.6(x^2 - 22x + 40) \\ &= -2.6(x - 2)(x - 20) \end{aligned}$$

Since the coefficient $-2.6$ is negative, it follows that $P(x) > 0$ only if the terms $(x - 2)$ and $(x - 20)$ have different signs; that is, when $x - 2 > 0$ and $x - 20 < 0$. Thus, production is profitable for $2 < x < 20$.

Correct answer is: Production is profitable for $2 < x < 20$.

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4.

**Suppose the total cost in dollars of manufacturing $q$ units of a certain commodity is given by the function $C(q) = q^3 - 60q^2 + 500q + 100$. Compute the cost of manufacturing 3 units of the commodity.**

**Cost of 3 units = $ ____**

Solution:

The cost of manufacturing 3 units is the value of the total cost function when $q = 3$. That is,

$$\begin{aligned} \text{Cost of 3 units} &= C(3) \\ &= (3)^3 - 60(3)^2 + 500(3) + 100 \\ &= \$1087 \end{aligned}$$

Correct answer is: 1087

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5.

**Suppose the total cost in dollars of manufacturing $q$ units of a certain**

**commodity is given by the function $C(q) = q^3 - 10q^2 + 500q + 200$. Compute the cost of manufacturing the 6th unit of the commodity.**

**Cost of 6th unit = $ ____**

Solution:

The cost of manufacturing 6 units is the value of the total cost function when $q = 6$. That is,

$$\begin{aligned} \text{Cost of 6 units} &= C(6) \\ &= (6)^3 - 10(6)^2 + 500(6) + 200 \\ &= 3056 \end{aligned}$$

The cost of manufacturing the 6th unit is the difference between the cost of manufacturing 6 units and the cost of manufacturing 5 units. That is,

Cost of 6th unit = $C(6) - C(5) = 3056 - 2575 = \$481$

Correct answer is: 481

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6.

**Find the composite function $f(g(x))$, where $f(u) = u^2 + 2u + 5$ and $g(x) = x + 1$.**

$f(g(x))$ = _____

Solution:

Replace $u$ by $x + 1$ in the formula for $f(u)$ to get

$$\begin{aligned} f(g(x)) &= (x+1)^2 + 2(x+1) + 5 \\ &= (x^2 + 2x + 1) + (2x + 2) + 5 \\ &= x^2 + 4x + 8 \end{aligned}$$

Correct answer is: $f(g(x)) = 4x + 8 + x^2$

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7.

**If $f(x) = \dfrac{5}{x-3} + 4(x-3)^3$, find functions $g(u)$ and $h(x)$ such that $f(x) = g(h(x))$.**

$g(u)$= _____ and $h(x)$= _____

Solution:

The form of the given function is

$f(x) = \dfrac{5}{\square} + 4(\square)^3$

where each box contains the expression $x - 3$. Thus, $f(x) = g(h(x))$, where

$g(u) = \dfrac{5}{u} + 4u^3$ and $h(x) = x - 3$

*outer function* *inner function*

Correct answer is: $g(u)=u^3 4 + \dfrac{5}{u}$ and $h(x)=x - 3$

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8.

**Find the composite function $f(g(x))$.**

**$f(u) = 7u^2 + 3u - 3$ and $g(x) = x + 10$**

$f(g(x))$ = _____

Solution:

$$f(g(x)) = f(x + 10)$$
$$= 7(x + 10)^2 + 3(x + 10) - 3 = 7x^2 + 143x + 727.$$

Correct answer is: $f(g(x)) = 7x^2 + 143x + 727$

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9.

**A difference quotient is an expression of the general form**

$$\frac{f(x+h)-f(x)}{h}$$

**where *f* is a given function of *x* and *h* is a number.**

**Find the difference quotient for $f(x) = 4x - 9x^2$.**

Solution:

For $f(x) = 4x - 9x^2$,

$$\frac{f(x+h)-f(x)}{h} = \frac{[4(x+h) - 9(x+h)^2] - [4x - 9x^2]}{h}$$

$$= \frac{[4x + 4h - 9x^2 - 18hx - 9h^2 - 4x + 9x^2]}{h}$$

$$= \frac{4h - 18hx - 9h^2}{h}$$

$$= 4 - 18x - 9h$$

Correct answer is: $-18x + 4 - 9h$

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10.

**Find the indicated function.**

**$f(x + 53)$ where $f(x) = \frac{x - 94}{x}$**

$f(x + 53) = \frac{\_\_\_\_\_}{\_\_\_\_\_}$

Solution:

$$f(x+53) = \frac{(x+53) - 94}{x+53}$$

$$= \frac{x - 41}{x + 53}$$

Correct answer is: $f(x + 53) = \frac{x - 41}{x + 53}$

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11.

**At a certain factory, the total cost of manufacturing $q$ units during the daily production run is $C(q) = q^2 + q + 400$ dollars. On a typical workday, $q(t) = 10t$ units are manufactured during the first $t$ hours of a production run.**

**When will the total manufacturing cost reach $4060?**

**Your Answer: ____ hours**

Solution:

$C(q) = q^2 + q + 400$ and $q(t) = 10t$,

thus $C[q(t)] = C(10t) = (10t)^2 + 10t + 400 = 100t^2 + 10t + 400$.

$$100t^2 + 10t + 400 = 4060$$

$$100t^2 + 10t - 3660 = 0$$

Divide by 10 to get smaller numbers, then

$$10t^2 + t - 366 = (10t + 61)(t - 6) = 0 \text{ or } t = 6 \text{ hours.}$$

Disregard $t = -\frac{61}{10}$.

Correct answer is: 6

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12.

**Find the domain and range of the function $f(x) = \frac{1}{x - 5}$.**

**A.** **The domain is the set of all numbers $x \neq 5$.**
**The range is the set of all numbers $y \neq 0$.**

**B.** **The domain is the set of all numbers $x > 5$.**
**The range is the set of all numbers $y > 0$.**

**C.** **The domain is the set of all numbers $x \neq 0$.**
**The range is the set of all numbers $y \neq 5$.**

**D.** **The domain is the set of all numbers $x < 5$.**
**The range is the set of all numbers $y \neq 0$.**

Solution:

Since division by any number other than 0 is possible, the domain of *f* is the set of all numbers $x \neq 5$.

The range of *f* is the set of all numbers *y* except 0, since for any $y \neq 0$,

there is an *x* such that $y = \frac{1}{x - 5}$; in particular, $x = 5 + \frac{1}{y}$.

Correct answer is: A

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13.

**Find the domain of $g(t) = \sqrt{t + 5}$.**

All real numbers *t* for which ____

Solution:

Since negative numbers do not have real square roots, $g(t)$ can be evaluated only when $t + 5 \geq 0$, so the domain of *g* is the set of all numbers *t* such that $t \geq -5$.

Correct answer is: All real numbers *t* for which $t \geq -5$

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14.

**Find the domain of the function $f(t) = \frac{t + 2}{t^2 + 9t + 18}$.**

**A.** **The domain is the set of all numbers $t \neq -3$ and $t \neq -6$.**

**B.** **The domain is the set of all numbers $t > -3$ and $t < -6$.**

**C.** **The domain is the set of all numbers $t \neq 3$ and $t \neq 6$.**

**D.** **The domain is the set of all numbers $x \neq 0$.**

Solution:

$$f(t) = \frac{t + 2}{t^2 + 9t + 18}$$

$$t^2 + 9t + 18 = (t + 3)(t + 6) \neq 0$$

if $t \neq -3$ and $t \neq -6$.

Correct answer is: A

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15.

**Find the domain of $g(x) = \sqrt{4x - 4}$.**

All real numbers $x$ for which _____ .

Solution:

Since negative numbers do not have real square roots, the domain is all real numbers such that

$4x - 4 \geq 0$,

or $x \geq 1$.

Correct answer is: All real numbers $x$ for which $x \geq 1$.