MATH 234 BL1 LECTURE 02 NOTES

SECTION 1.3: LINEAR FUNCTIONS

1. Lines

A **linear function** is a function of the form $f(x) = ax + b$, for constants a and b. When graphed, such a function is a line, usually written $y = ax+b$, called the **slope-intercept form**. Notice that when $x = 0$, $y = a(0)+b = b$, so the point $(0, b)$ is on the line and is called the **y-intercept**. The constant a is called the **slope** and is the rise of the graph over the run of the graph. For any two points (x_1, y_1) and (x_2, y_2), we have that

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1},$$

where Δ means "change in", so Δy = change in $y = y_2 - y_1$.

A horizontal line has slope zero, so its equational form is $y = b$ for some constant b since all points on the line are of the form (x, b). Similarly, a vertical line has equational form $x = a$ since all the points on the line are of the form (a, y).

Examples 1. See the graphs drawn on the board.

1.1. **Point-Slope Equation.** From the slope equation, we can generate another equation that is extremely useful. Suppose we know the slope of a line is m and we also know a point (x_1, y_2) on the line. For any other point (x, y) on the line, we can write

$$m = \frac{y - y_1}{x - x_1}$$

and rearranging, we find that

$$y - y_1 = m(x - x_1).$$

This is called the **point-slope equation**.

Example 2. Suppose we have a line of slope $\frac{1}{2}$ and the point $(2, 1)$ lies on the line. Does the line pass through the origin $(0, 0)$?

Solution: First, we use the point slope equation to write the equation of the line: $y - 1 = \frac{1}{2}(x - 2)$. To see if $(0, 0)$ lies on the line, let $x = 0$; then $y - 1 = \frac{1}{2}(0 - 2) = -1$, so $y = -1 + 1 = 0$ and we conclude that the line passes through the origin.

1.2. **Parallel and Perpendicular Lines.** Parallel lines have the same slope, as can be seen from the graph of any two parallel lines, but will have different intercepts (unless they are exactly the same line). Less obvious is the fact that perpendicular lines have negative-reciprocal slopes; in equations,

$$m_{\perp} = \frac{-1}{m_{||}}.$$

Example 3. Find the equation of the line perpendicular to the line from the previous example that passes through the origin.

Solution: The previous line was $y - 1 = \frac{1}{2}(x - 2)$. We know that both lines pass through the origin, so we only need the slope of the perpendicular line to write out its equation. It is

$$m = \frac{-1}{\frac{1}{2}} = -2,$$

so the equation of the line (using the point-