Version A

Lecture 04 Solutions

Solutions

1.

Find the derivative of the function $f(x) = 18x^2$.

$f'(x) =$ _____

Solution:

The difference quotient for $f(x)$ is

$$\frac{f(x+h) - f(x)}{h} = \frac{18(x+h)^2 - 18x^2}{h}$$

$$= \frac{18(x^2 + 2hx + h^2) - 18x^2}{h}$$

$$= \frac{36hx + 18h^2}{h}$$ *combine terms*

$$= 36x + 18h$$ *cancel common h terms*

Thus, the derivative of $f(x) = 18x^2$ is the function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (36x + 18h)$$

$$= 36x$$

Correct answer is: $f'(x) = 36x$

2.

Compute the derivative of the function $f(x) = 6x - 2$.

$f'(x) =$ _____

Solution:

The difference quotient for $f(x)$ is

$$\frac{f(x+h) - f(x)}{h} = \frac{6(x+h) - 2 - (6x-2)}{h}$$

$$= \frac{6x + 6h - 2 - 6x + 2}{h}$$

$$= \frac{6h}{h} \qquad \textit{combine terms}$$

$$= 6 \qquad \textit{cancel common h terms}$$

Thus, the derivative of $f(x) = 6x - 2$ is the function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \blacksquare 0} (6)$$

$$= 6$$

Correct answer is: $f'(x) = 6$

3.

Compute the derivative of the given function and find the equation of the lin that is tangent to its graph for the specified value $x = c$.

$f(x) = \frac{9}{x}$; $c = 17$

$f'(x) =$ _____ and the equation at $x = 17$ is $y =$ _____

Solution:

If $f(x) = \frac{9}{x}$, then $f(x+h) = \frac{9}{x+h}$.

The difference quotient (DQ) is

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{9}{(x+h)} - \frac{9}{x}}{h}$$

$$= \frac{\frac{9}{(x+h)} - \frac{9}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= \frac{9x - 9(x+h)}{h(x)(x+h)}$$

$$= \frac{-9}{x(x+h)}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = -\frac{9}{x^2}.$$

The slope of the line is $m = f'(17) = \frac{-9}{289}$.

Since $f(17) = \frac{9}{17}$,

$(17, \frac{9}{17})$ is a point on the curve and the equation of the tangent line is

$$y - \frac{9}{17} = \frac{-9}{289}(x - 17)$$

$$y = -\frac{9}{289}x + \frac{18}{17}$$

Correct answer is: $f'(x) = -\frac{9}{x^2}$ and the equation at $x = 17$ is $y = -\frac{9}{289}x + \frac{18}{17}$

4.

Let $f(x) = 5x^2$.

Use calculus to compute the slope of the line that is tangent to the graph when $x = -6$.

$m =$ _____

Solution:

$$f'(-6) = \lim_{h \to 0} \frac{f(-6+h) - f(-6)}{h}$$

$$= \lim_{h \to 0} \frac{5(-6+h)^2 - 5(-6)^2}{h}$$

$$= \lim_{h \to 0} (-60 + 5h)$$

$$= -60$$

Correct answer is: $m = -60$