Version A

Lecture 05 Solutions

Solutions

1.

For the function

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x < 2 \\ 7x + 3 & \text{if } x \geq 2 \end{cases}$$

evaluate the one-sided limit $\lim_{x \to 2^-} f(x)$**.**

$\lim_{x \to 2^-} f(x) =$ _____

Solution:

The graph of $f(x)$ is shown in the figure below.

The graph of $f(x) = \begin{cases} 4 - x^2 & \text{if } x < 2 \\ 7x + 3 & \text{if } x \geq 2 \end{cases}$

Since $f(x) = 4 - x^2$ for $x < 2$, we have

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (4 - x^2) = 0$$

Correct answer is: 0

2.

Find the indicated one-sided limit.

$$\lim_{x \to 2^+} (4x^2 - 6) = \_\_\_\_\_$$

Solution:

$$\lim_{x \to 2^+} (4x^2 - 6) = 4(2)^2 - 6 = 10$$

Correct answer is: 10

3.

Is the polynomial $p(x) = 7x^3 - x + 9$ continuous at $x = 2$?

- **A. The polynomial is discontinuous at $x = 2$.**
- **B. The polynomial is continuous at $x = 2$.**

Solution:

Verify that the three criteria for continuity are satisfied. Clearly $p(2)$ is defined; in fact,
$p(2) = 63$. Moreover, $\lim_{x \to 2} p(x)$ exists and $\lim_{x \to 2} p(x) = 63$. Thus,

$$\lim_{x \to 2} p(x) = p(2)$$

as required for $p(x)$ to be continuous at $x = 2$.

Correct answer is: B

4.

Discuss the continuity of the following function:

$$f(x) = \frac{2}{x}$$

A. $f(x)$ **is continuous for all real numbers** x**.**

B. $f(x)$ **is continuous for all** $x \neq 2$**.**

C. $f(x)$ **is continuous for all** $x \neq -2$**.**

D. $f(x)$ **is continuous for all** $x \neq 0$**.**

Solution:

The function is rational and is therefore continuous wherever it is defined (that is, wherever its denominator is not zero).

$f(x) = \frac{2}{x}$ is defined everywhere except $x = 0$, and thus it is continuous for all $x \neq 0$.

____Continuous for $x \neq 0$

Correct answer is: D

5.

Discuss the continuity of the following function:

$$h(x) = \begin{cases} x + 9 & \text{if } x < 5 \\ 2 - x & \text{if } x \geq 5 \end{cases}$$

A. ***h*(*x*) is continuous at every number *x* other than 0.**

B. ***h*(*x*) is continuous at every number *x* other than 14.**

C. ***h*(*x*) is continuous at every number *x* other than 5.**

D. ***h*(*x*) is continuous for all real numbers *x*.**

Solution:

This function is defined in two pieces. First check for continuity at $x = 5$, the value of x that separates the two pieces. You find that $\lim_{x \to 5} h(x)$ does not exist, since $h(x)$ approaches 14 from the left and −3 from the right. Thus, $h(x)$ is not continuous at 5.

Continuous for $x \neq 5$

However, since the polynomial $x + 9$ and $2 - x$ are each continuous for every value of x, it follows that $h(x)$ is continuous at every number x other than 5.

Correct answer is: C

6.

For what value of the constant *A* is the following function continuous for all real *x*?

$f(x) = \begin{cases} Ax + 16 & \text{if } x < 2 \\ x^2 - 3x + 8 & \text{if } x ■ 2 \end{cases}$

***f* is continuous for all *x* only when *A* = _____ .**

Solution:

Since $Ax + 16$ and $x^2 - 3x + 8$ are both polynomials, it follows that $f(x)$ will be continuous everywhere except possibly at $x = 2$. Moreover, $f(x)$ approaches $2A + 16$ as x approaches 2 from the left and approaches 6 as x approaches 2 from the right. Thus, for $\lim_{x \to 2} f(x)$ to exist, we must have $2A + 16 = 6$ or $A = -5$, in which case

$$\lim_{x \to 2} f(x) = 6 = f(2)$$

This means that f is continuous for all x only when $A = -5$.

Correct answer is: -5
