Version A

Lecture 06 Solutions

Solutions

1.

Differentiate the polynomial $y = 5x^3 - 2x^2 + 15x - 2$.

$\frac{dy}{dx} = $ _____

Solution:

Differentiate this sum term by term to get

$$\frac{dy}{dx} = \frac{d}{dx}[5x^3] + \frac{d}{dx}[-2x^2] + \frac{d}{dx}[15x] + \frac{d}{dx}[-2]$$

$$= 15x^2 - 4x^1 + 15x^0 + 0 \qquad \textit{recall } x^0 = 1$$

$$= 15x^2 - 4x + 15$$

Correct answer is: $\frac{dy}{dx} = 15x^2 - 4x + 15$

2.

Differentiate the given function.

$f(x) = \frac{1}{3}x^8 - \frac{1}{5}x^4 - 14x + 7$.

$f'(x) = $ _____

Solution:

$f(x) = \frac{1}{3}x^8 - \frac{1}{5}x^4 - 14x + 7$

$f'(x) = 8 \cdot \frac{1}{3}x^7 - 4 \cdot \frac{1}{5}x^3 - 14 + 0$

$= \frac{8}{3}x^7 - \frac{4}{5}x^3 - 14$

Correct answer is: $f'(x) = \frac{8}{3}x^7 - \frac{4}{5}x^3 - 14$

3.

Find the equation of the line that is tangent to the graph of the given function at the point (c, $f(c)$) for the specified value of $x = c$.
$f(x) = x^4 - 4x^3 + 4x^2 - 11$; $x = 1$

$y =$ _____

Solution:

$f'(x) = 4x^3 - 12x^2 + 8x$

$f(1) = 1 - 4 + 4 - 11 = -10$ so $(1, -10)$ is a point on the tangent line.

The slope is $m = f'(1) = 4 - 12 + 8 = 0$.

The equation of the tangent line is

$y - (-10) = 0(x - 1)$

or $y = 0x - 10$.

Correct answer is: $y = 0x - 10$

4.

Suppose a person standing at the top of a building 432 feet high throws a ball vertically upward with an initial velocity of 96 ft/sec. When is the

velocity 0? What is the significance of this time?

- **A. The ball is at its lowest point when $t = 6$ seconds.**
- **B. The ball is at its highest point when $t = 4$ seconds.**
- **C. The ball is at its highest point when $t = 3$ seconds.**
- **D. The ball is at its lowest point when $t = 2$ seconds.**

Solution:

Since $g = 32$, $V_0 = 96$, and $H_0 = 432$, the height of the ball above the ground at time t is

$$H(t) = -16t^2 + 96t + 432 \text{ feet}$$

The velocity at time t is

$$v(t) = \frac{dH}{dt} = -32t + 96 \text{ (ft/sec)}$$

The velocity is zero when $v(t) = -32t + 96 = 0$, which occurs when $t = 3$ seconds. For $t < 3$, the velocity is positive and the ball is rising, and for $t > 3$, the ball is falling. Thus, the ball is at its highest point when $t = 3$ seconds.

Correct answer is: C

5.

Differentiate the product $P(x) = (x - 9)(4x - 1)$ by expanding $P(x)$ and using the polynomial rule.

$P'(x) =$ _____

Solution:

We have $P(x) = 4x^2 - 37x + 9$,
so $P'(x) = 8x - 37$.

Correct answer is: $P'(x) = 8x - 37$

6.

Differentiate the product $P(x) = (x - 9)(2x - 11)$ by the product rule.

$P'(x) =$ _____

Solution:

By the product rule

$$P'(x) = (x-9)\frac{d}{dx}[2x-11] + (2x-11)\frac{d}{dx}[x-9]$$

$$= (x-9)(2) + (2x-11)(1) = 4x - 29$$

Correct answer is: $P'(x) = 4x - 29$

7.

Differentiate the given function.

$$y = \frac{2x-3}{9x+4}$$

$\frac{dy}{dx} = \frac{\_\_\_\_\_}{\_\_\_\_\_}$

Solution:

$$\frac{dy}{dx} = \frac{(9x+4)\frac{d}{dx}(2x-3) - (2x-3)\frac{d}{dx}(9x+4)}{(9x+4)^2}$$

$$= \frac{2(9x+4) - 9(2x-3)}{(9x+4)^2}$$

- = 35

$(9x+4)^2$

Correct answer is: $\frac{dy}{dx} = \frac{35}{(9x+4)^2}$

8.

Find an equation for the tangent line to the given curve at the point where *x*
Simplify your answer.
$y = \frac{9x}{6x+7}$; $x_0 = 1$

$y =$ _____

Solution:

$\frac{dy}{dx} = \frac{63}{(6x+7)^2}$

When $x = 1$, $y = \frac{9}{13}$ and $\frac{dy}{dx} = \frac{63}{169}$.

The equation of the tangent line at $\left(1\ ,\ \frac{9}{13}\right)$ is

$y - \frac{9}{13} = \frac{63}{169}(x-1)$, or

$y = \frac{63}{169}x + \frac{54}{169}$.

Correct answer is: $y = \frac{63}{169}x + \frac{54}{169}$
