Version A

Lecture 07 Solutions

Solutions

1.

If the position of an object moving along a straight line is given by $s(t) = 8t^3 - 8t^2 + 6t$ at time t, find its acceleration.

$a(t) =$ ____

Solution:

The velocity of the object is

$$v(t) = \frac{ds}{dt} = 24t^2 - 16t + 6$$

and its acceleration is

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 48t - 16$$

Correct answer is: $a(t) = 48t - 16$

2.

Find the fifth derivative of the function:

$$f(x) = 5x^3 + 2x^2 + 2x - 5$$

$f^{(5)}(x) =$ ____

Solution:

$f'(x) = 15x^2 + 4x + 2$

$f''(x) = 30x + 4$

$f'''(x) = 30$

$f^{(4)}(x) = 0$

$f^{(5)}(x) = 0$

Correct answer is: $f^{(5)}(x) = 0$

3.

The gross domestic product (GDP) of a certain country was $N(t) = t^2 + 7t + 118$ billion dollars t years after 1991. At what rate was the GDP changing with respect to time in 2003?

Your Answer: ____ billion dollars per year.

Solution:

The rate of change of the GDP is the derivative $N'(t) = 2t + 7$.

The rate of change in 2003 was $N'(12) = 2(12) + 7 = 31$ billion dollars per year.

Correct answer is: 31

4.

The gross domestic product (GDP) of a certain country was $N(t) = t^2 + 7t + 102$ billion dollars t years after 1990. At what percentage rate was the GDP changing with respect to time in 2001? Round your answer to the nearest integer.

The percentage rate of change of the GDP in 2001 was ____ % per year.

Solution:

The rate of change of the GDP in the derivative $N'(t) = 2t + 7$.

The rate of change in 2001 was $N'(11) = 2(11) + 7 = 29$ billion dollars per year.

The percentage rate of change of the GDP in 2001 was

$$100 \frac{N'(11)}{N(11)} = 100 \frac{29}{300} \approx 10\% \text{ per year}$$

Correct answer is: 10

5.

Find $\frac{dy}{dx}$ if $y = (x^2 + 6)^3 - 3(x^2 + 6)^2 + 3$.

$\frac{dy}{dx} =$ ____

Solution:

Note that $y = u^3 - 3u^2 + 3$, where $u = x^2 + 6$. Thus,

$$\frac{dy}{du} = 3u^2 - 6u \quad \text{and} \quad \frac{du}{dx} = 2x$$

and according to the chain rule,

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (3u^2 - 6u)(2x)$$

Express the derivative in terms of x alone by substituting $x^2 + 6$ for u as follows:

$$\frac{dy}{dx} = [(3u^2 - 6u)](2x) = [3(x^2 + 6)^2 - 6(x^2 + 6)](2x) \quad \textit{replace u with } x^2 + 6$$

$$= 6x(x^2 + 6)[(x^2 + 6) - 2] \quad \textit{factor out } 6x(x + 6)$$

$$= 6x(x^2 + 6)(x^2 + 4) \quad \textit{combine terms in the brackets}$$

Correct answer is: $\frac{dy}{dx} = 6x(x^2 + 6)(x^2 + 4)$

6.

Differentiate the function $f(x) = \dfrac{1}{(6x+7)^4}$.

$f'(x) =$ _____

Solution:

Although you can use the quotient rule, it is easier to rewrite the function as

$$f(x) = (6x+7)^{-4}$$

and apply the general power rule to get

$$f'(x) = \left[-4(6x+7)^{-5}\right]\frac{d}{dx}[6x+7] = -4(6x+7)^{-5}(6) = -\frac{24}{(6x+7)^5}$$

Correct answer is: $f'(x) = -\dfrac{24}{(6x+7)^5}$

7.

Differentiate the function $f(x) = (9x+7)^3(3x-2)^4$ **and simplify your answer.**

$f'(x) =$ _____

Solution:

First apply the product rule to get

$$f'(x) = (9x+7)^3\frac{d}{dx}[(3x-2)^4] + (3x-2)^4\frac{d}{dx}[(9x+7)^3]$$

Continue by applying the general power rule to each term:

$$\begin{aligned} f'(x) &= (9x+7)^3[4(3x-2)^3(3)] + (3x-2)^4[3(9x+7)^2(9)] \\ &= 12(9x+7)^3(3x-2)^3 + 27(3x-2)^4(9x+7)^2 \end{aligned}$$

Finally, simplify your answer by factoring:

$$f'(x) = 3(9x+7)^2(3x-2)^3[4(9x+7)+9(3x-2)]$$
$$= 3(9x+7)^2(3x-2)^3[36x+28+27x-18]$$
$$= 3(9x+7)^2(3x-2)^3(63x+10)$$

Correct answer is: $f'(x) = 3(9x + 7)^2(3x - 2)^3(63x + 10)$

8.

Differentiate $f(x) = \dfrac{(x+4)^5}{(4-x)^4}$ and simplify your answer.

$f'(x) =$ _____

Solution:

$$f(x) = \frac{(x+4)^5}{(4-x)^4}$$

$$f'(x) = \frac{(4-x)^4 \frac{d}{dx}(x+4)^5 - (x+4)^5 \frac{d}{dx}(4-x)^4}{[(4-x)^4]^2}$$

Now,

$$\frac{d}{dx}(x+4)^5 = 5(x+4)^4 \frac{d}{dx}(x+4)$$
$$= 5(x+4)^4$$

and

$$\frac{d}{dx}(4-x)^4 = 4(4-x)^3 \frac{d}{dx}(4-x)$$
$$= -4(4-x)^3$$

So,

$$f'(x) = \frac{5(4-x)^4(x+4)^4 + 4(x+4)^5\,(4-x)^3}{(4-x)^8}$$

$$= \frac{(4-x)^3(x+4)^4(5(4-x)+4(x+4))}{(4-x)^8}$$

$$= \frac{(x+4)^4(36-x)}{(4-x)^5}$$

Correct answer is: $f'(x) = \dfrac{(-x+36)(x+4)^4}{(-x+4)^5}$

9.

Find an equation of the line that is tangent to the graph of $f(x) = \left(\dfrac{x+8}{x-8}\right)^3$ fo

$y =$ _____

Solution:

$$f(x) = \left(\frac{x+8}{x-8}\right)^3$$

$$f'(x) = 3\left(\frac{x+8}{x-8}\right)^2\left(\frac{(x-8)(1)-(x+8)}{(x-8)^2}\right)$$

At $x = 4$, $y = f(4) = -27$, $f'(4) = -27$ and the equation of the tangent line is $y + 27 = -27(x - 4)$ or $y = -27x + 81$.

Correct answer is: $y = -27x + 81$

10.

Find the second derivative of $f(t) =$ 3 .

$3t + 9$

$f''(t) =$ ____

Solution:

$$f(t) = \frac{3}{3t+9} = 3(3t+9)^{-1}$$

$$f'(t) = -3(3t+9)^{-2}(3) = -\frac{9}{(3t+9)^2}$$

$$f''(t) = (-2)(-9)(3t+9)^{-3}(3) = \frac{54}{(3t+9)^3}$$

Correct answer is: $f''(t) = \dfrac{54}{(3t+9)^3}$
