Version A

Lecture 10 Solutions

Solutions

1.

Determine where the graph of the given function is concave upward and concave downward. Find the coordinates of the inflection point(s).

$$f(x) = x^3 + 18x^2 + 3x + 18$$

(____ , ____):inflection point

Solution:

$f'(x) = 3x^2 + 36x + 3$

$f''(x) = 6x + 36 = 6(x + 6)$

$f''(x) = 0$ when $x = -6$.

When $x < -6$, $f''(x) < 0$ so f is concave down.

$x > -6$, $f''(x) > 0$ so f is concave up.

Since the concavity changes at the critical value $x = -6$,
the point (−6, 432) is an inflection point.

Correct answer is: (− 6,432):inflection point

2.

Determine where the graph of $f(x) = 4(x - 3)^{7/3}$ is concave downward. Find the coordinates of all inflection points. Be sure to use commas to separate the end points of intervals and the coordinates of points.

$f''(x) < 0$ for $x \in$ (____) ; Inflection point (____)

Solution:

$$f(x) = 4(x-3)^{7/3}$$

$$f'(x) = \frac{28}{3}(x-3)^{4/3}$$

$$f''(x) = \frac{112}{9}(x-3)^{1/3} = 0$$

when $x = 3$. $f(3) = 0$.

$$\begin{array}{ccc} - & & + \\ \longleftarrow & \vert & \longrightarrow \\ & 3 & \end{array}$$

Thus (3, 0) is an inflection point.

Correct answer is: $f''(x) < 0$ for $x \in \left(-\infty, 3\right)$; Inflection point $\left(3, 0\right)$

3.

Use the second derivative test to find the relative maxima and minima of the function $f(x) = x^3 + 12x^2 + 7$.

Rel. maximum: (_____ , _____). Rel. minimum: (_____ , _____).

Solution:

Since the first derivative

$$f'(x) = 3x^2 + 24x = 3x(x+8)$$

is zero when $x = 0$ and $x = -8$, the corresponding points (0, 7) and (−8, 263) are the critical points of f. To test these points, compute the second derivative

$$f''(x) = 6x + 24$$

and evaluate it at $x = -8$ and $x = 0$. Since

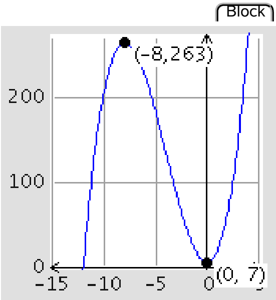
$$f''(-8) = -24 < 0$$

it follows that the critical point (−8, 263) is a relative maximum, and since

$$f''(0) = 24 > 0$$

it follows that the critical point (0, 7) is a relative minimum.

The graph is shown below.



The graph of $f(x) = x^3 + 12x^2 + 7$.

Correct answer is: Rel. maximum: (– 8, 263). Rel. minimum: (0, 7).

4.

Use the second derivative test to find the relative maxima and minima of the given function.

$$f(x) = 7(x^2 - 4)^2$$

Rel. max: (_____ , _____). Rel. min: (_____ , _____) and (_____ , _____).

Solution:

Since the first derivative

$$f'(x) = 7(2)(2x)(x^2 - 4) = 28x(x + 2)(x - 2)$$

is zero when $x = 0$, $x = -2$, and $x = 2$, the corresponding points (0, 112), (–2, 0), and (2, 0) are the critical points of f. To test these points, compute the second derivative

$$f''(x) = 84x^2 - 112$$

and evaluate it at $x = 0$, $x = -2$, and $x = 2$. Since

$$f''(0) = -112 < 0$$

it follows that the critical point (0, 112) is a relative maximum, and since

$$f''(-2) = 224 > 0$$

it follows that the critical point (–2, 0) is a relative minimum, and since

$f''(2) = 224 > 0$

it follows that the critical point (2, 0) is a relative minimum.

Correct answer is: Rel. max: (0, 112). Rel. min: (– 2, 0) and (2, 0).

5.

A 6–year projection of population trends suggests that *t* years from now, the population of a certain community will be $P(t) = -t^3 + 12t^2 + 44t + 52$ thousand. At what time is the rate of population growth changing most rapidly?

That rate of population growth is most rapidly changing when *t* = ____ years.

Solution:

The rate of growth is the derivative of the population $P(t)$; that is,

$$R(t) = P'(t) = -3t^2 + 24t + 44$$

The derivative of the rate function is

$$R'(t) = P''(t) = -6t + 24$$

which is largest when $t = 0$. This means that rate of population growth is most rapidly changing when $t = 0$ years.

Correct answer is: 0
