Version A

Lecture 12 Solutions

Solutions

1.

If they exist, find the absolute maximum and absolute minimum of the function

$$f(x) = x^2 + \frac{250}{x} \text{ on the interval } x > 0.$$

Absolute minimum at (____ , ____).

Solution:

The function is continuous on the interval $x > 0$ since its only discontinuity occurs at $x = 0$. The derivative is

$$f'(x) = 2x - \frac{250}{x^2}$$

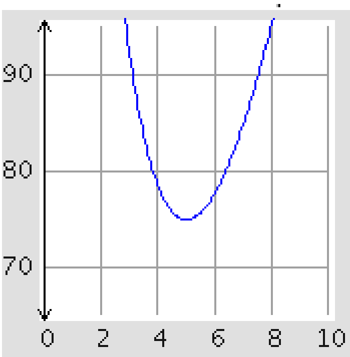
$$= \frac{2x^3 - 250}{x^2}$$

$$= \frac{2(x^3 - 125)}{x^2}$$

which is zero when

$x^3 - 125 = 0 \quad x^3 = 125 \quad \text{or} \quad x = 5$

Since $f'(x) < 0$ for $0 < x < 5$ and $f'(x) > 0$ for $x > 5$, the graph of f is decreasing for $0 < x < 5$ and increasing for $x > 5$, as shown below.



The function $f(x) = x^2 + \frac{250}{x}$

on the interval $x > 0$.

It follows that

$$f(5) = 5^2 + \frac{250}{5} = 75$$

is the absolute minimum of f on the interval $x > 0$ and that there is no absolute maximum.

Correct answer is: Absolute minimum at (5, 75).

2.

A manufacturer estimates that when q thousand units of a particular commodity are produced each month, the total cost will be $C(q) = 0.4q^2 + 6q + 35$ thousand dollars, and all q units can be sold at a price of $p(q) = 44.4 - 1.2q$ dollars per unit. Determine the level of production that results in maximum profit.

The maximum profit occurs when ____ thousand units are produced.

Solution:

The revenue is

$$R(q) = qp(q) = q(44.4 - 1.2q) = -1.2q^2 + 44.4q$$

thousand dollars, so the profit is

$$\begin{aligned} P(q) = R(q) - C(q) &= -1.2q^2 + 44.4q - (0.4q^2 + 6q + 35) \\ &= -1.6q^2 + 38.4q - 35 \end{aligned}$$

thousand dollars. We have

$$P'(q) = -1.6(2q) + 38.4$$
$$= -3.2q + 38.4$$
$$= 0$$

when

$$-3.2q + 38.4 = 0$$

$$q = \frac{38.4}{3.2} = 12$$

Since $P''(q) = -3.2$, it follows that $P''(12) < 0$, and the second derivative test tells us that maximum profit occurs when $q = 12$ (thousand) units are produced.

Correct answer is: 12

3.

A manufacturer estimates that when q thousand units of a particular commodity are produced each month, the total cost will be $C(q) = 0.4q^2 + 24q + 45$ thousand dollars, and all q units can be sold at a price of $p(q) = 177.6 - 1.2q$ dollars per unit. What is the maximum profit?

The maximum profit is $ ____ .

Solution:

The revenue is

$$R(q) = qp(q) = q(177.6 - 1.2q) = -1.2q^2 + 177.6q$$

thousand dollars, so the profit is

$$P(q) = R(q) - C(q) = -1.2q^2 + 177.6q - (0.4q^2 + 24q + 45)$$
$$= -1.6q^2 + 153.6q - 45$$

thousand dollars. We have

$$P'(q) = -1.6(2q) + 153.6$$
$$= -3.2q + 153.6$$
$$= 0$$

when

$$-3.2q + 153.6 = 0$$

$$q = \frac{153.6}{3.2} = 48$$

Since $P''(q) = -3.2$, it follows that $P''(48) < 0$, and the second derivative test tells us that maximum profit occurs when $q = 48$ (thousand) units are produced.

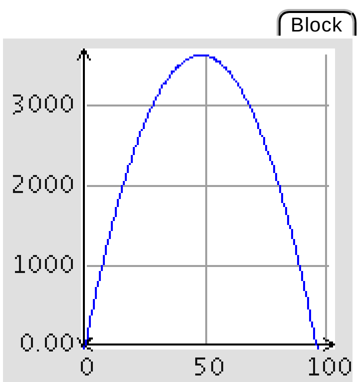
The maximum profit is

$$P(48) = -1.6(48)^2 + 153.6(48) - 45$$

$$= 3641.4$$

thousand dollars ($3641400).

The graph of the profit function is shown below.



The profit function
$P(q) = -1.6q^2 + 153.6q - 45.$

Correct answer is: 3641400

4.

Find the absolute maximum of the function

$$f(x) = 2x^3 + 12x^2 - 192x - 8$$

on the interval $-9 \le x \le 0$.

The absolute maximum of f is f (____) = ____ .

Solution:

From the derivative

$$f'(x) = 6x^2 + 24x - 192 = 6(x + 8)(x - 4)$$

you see that the critical numbers are $x = -8$ and $x = 4$. Of these, only $x = -8$ lies in the interval $-9 \leq x \leq 0$. Compute $f(x)$ at $x = -8$ and at the endpoints $x = -9$ and $x = 0$.

$$f(-8) = 1272 \quad f(-9) = 1234 \quad f(0) = -8$$

Compare these values to conclude that the absolute maximum of f on the interval $-9 \leq x \leq 0$ is $f(-8) = 1272$.

Correct answer is: The absolute maximum of f is $f(-8) = 1272$.

5.

Find the absolute maximum and absolute minimum of the given function on the specified interval.

$\boldsymbol{f(x) = x^2 + 14x + 1;\ -8 \leq x \leq -3}$

Absolute maximum: ____ , absolute minimum: ____

Solution:

$f'(x) = 2x + 14 = 2(x + 7)$

$f'(x) = 0$ when $x = -7$, which is in the interval

$f(-7) = -48$, $f(-8) = -47$ and $f(-3) = -32$.

So, $f(-3) = -32$ is the absolute maximum and $f(-7) = -48$ is the absolute minimum.

Correct answer is: Absolute maximum: -32, absolute minimum: -48

6.

Find the absolute minimum of the function

$$\boldsymbol{f(x) = (x^2 - 9)^5}$$

on the interval $-4 \le x \le 3$.

The absolute minimum of f is $f($ ____ $)$ = ____ .

Solution:

From the derivative

$$f'(x) = 5(x^2 - 9)^4 (2x) = 10x[(x + 3)(x - 3)]^4$$

you see that the critical numbers are $x = 0$, $x = -3$, and $x = 3$. All of these lie in the interval $-4 \le x \le 3$. Compute $f(x)$ at $x = 0$, $x = -3$, $x = 3$, and at the endpoint $x = -4$.

$f(0) = -59049$

$f(-3) = 0$

$f(3) = 0$

$f(-4) = 16807$

Compare these values to conclude that the absolute minimum of f on the interval $-4 \le x \le 3$ is $f(0) = -59049$.

Correct answer is: The absolute minimum of f is $f(0) = -59049$.

7.

A manufacturer estimates that when q thousand units of a particular commodity are produced each month, the total cost will be $C(q) = 0.2q^2 + 2q + 80$ thousand dollars, and all q units can be sold at a price of $p(q) = 22.2 - 1.2q$ dollars per unit. At what level of production is the average cost per unit

$$A(q) = \frac{C(q)}{q}$$

minimized? What is the minimal average cost?

With production ____ thousand units the average cost is ____ $\frac{\text{dollars}}{\text{unit}}$.

Solution:

The average cost is

$$A(q) = \frac{C(q)}{q} = \frac{0.2q^2 + 2q + 80}{q} \qquad \frac{\textit{thousand dollars}}{\textit{thousand units}}$$

$$= 0.2q + 2 + \frac{80}{q} \qquad \frac{\textit{dollars}}{\textit{units}}$$

for $q > 0$ (the level of production cannot be negative or zero). You find

$$A'(q) = 0.2 - \frac{80}{q^2} = \frac{0.2q^2 - 80}{q^2}$$

which is 0 for $q > 0$ only when $q = 20$. Since

$$A''(q) = \frac{160}{q^3} > 0 \text{ when } q > 0$$

it follows from the second derivative test for absolute extrema that average cost $A(q)$ is minimized when $q = 20$ (thousand) units. The minimal average cost is

$$A(20) = 0.2(20) + 2 + \frac{80}{20} = 10 \qquad \textit{dollars/unit}$$

Correct answer is: With production 20 thousand units the average cost is 10 $\frac{\text{dollars}}{\text{unit}}$.

8.

A manufacturer estimates that when q thousand units of a particular commodity are produced each month, the total cost will be $C(q) = 0.2q^2 + 8q + 80$ thousand dollars, and all q units can be sold at a price of $p(q) = 22.2 - 1.2q$ dollars per unit. At what level of production is the average cost equal to the marginal cost $C'(q)$?

The average cost is equal to the marginal cost when q = ____ thousand units.

Solution:

The average cost is

$$A(q) = \frac{C(q)}{q} = \frac{0.2q^2 + 8q + 80}{q} \qquad \frac{\textit{thousand dollars}}{\textit{thousand units}}$$

$$= 0.2q + 8 + \frac{80}{q} \qquad \frac{dollars}{units}$$

The marginal cost is $C'(q) = 0.4q + 8$, and it equals average cost when

$$0.4q + 8 = 0.2q + 8 + \frac{80}{q}$$

$$0.2q = \frac{80}{q}$$

$$0.2q^2 = 80$$

$$q = 20 \text{ (thousand) units}$$

The graphs of the marginal cost $C'(q)$ and average cost $A = \dfrac{C(q)}{q}$ are shown below.

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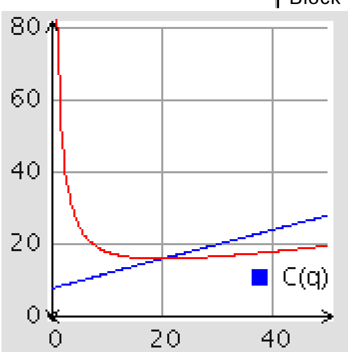

Average and marginal cost

Correct answer is: 20

