Version A

Lecture 14,15 Solutions

Solutions

1.

Find all real numbers *x* that satisfy the equation $4^{24 - x} = 16^{x}$.
x **= ____**

Solution:

$4^{24 - x} = 16^{x}$

$4^{24 - x} = (4^{2})^{x}$

$4^{24 - x} = 4^{2x}$

By the equality rule of exponential functions,
$24 - x = 2x$, or $x = 8$.

Correct answer is: 8

2.

Find the values of the constants *C* and *b* so that the curve $y = Cb^{x}$ contains the points (3, 4) and (4, 16). Express your answers as integers or simplified fractions.

C = ____ ; *b* = ____

Solution:

For $y = Cb^{x}$ to contain (3, 4) and (4, 16) we must have $4 = Cb^{3}$ and $16 = Cb^{4}$.

Dividing the second equation by the first gives

$$\frac{16}{4} = 4 = \frac{Cb^4}{Cb^3} = b.$$

Substituting $b = 4$ into the first equation gives

$$4 = 4^3C = 64C \text{ or } C = \frac{1}{16} = \frac{1}{16}.$$

Thus $y = \frac{1}{16}(4^x)$.

Correct answer is: $C = \frac{1}{16}$; $b = 4$

3.

Suppose $1400 is invested at an annual interest rate of 6%. Compute the balance after 8 years if the interest is compounded continuously.
Round your answer to the nearest cent.

The balance after 8 years is $ ____ .

Solution:

For continuously compounded interest use the formula $B(t) = Pe^{rt}$, with $t = 8$, $P = 1400$, and $r = 0.06$:

$$B(8) = 1400e^{0.48} \approx \$2262.50$$

This value, $2262.50, is an upper bound for the possible balance. No matter how often interest is compounded, $1400 invested at an annual interest rate of 6% cannot grow to more than $2262.50 in 8 years.

Correct answer is: 2262.5

4.

Suppose $9000 is invested at an annual interest rate of 7%. Compute the

balance after 14 years if the interest is compounded semi-annually. Round your final answer to the nearest cent.

Your Answer: $ ____

Solution:

$$B(t) = P\left(1 + \frac{r}{k}\right)^{kt} \text{ dollars.}$$

With semi-annual compounding $k = 2$.

$$B(14) = 9000\left(1 + \frac{0.07}{2}\right)^{2 \cdot 14}$$

$$= (9000)(2.6202)$$

$$= \$23581.55$$

Correct answer is: 23581.55

5.

Suppose $6000 is invested at an annual interest rate of 4%. Compute the balance after 10 years if the interest is compounded continuously. Round your final answer to the nearest cent.

Your Answer: $ ____

Solution:

With semi-annual compounding,

$$B(t) = Pe^{rt}.$$

$$B(t) = (6000)e^{(0.04)(10)}.$$

$$= (6000)e^{0.4}$$

$$= \$8950.95$$

Correct answer is: 8950.95

6.

Evaluate

$\log_2\left(\frac{1}{256}\right) =$ ____

Solution:

$\log_2 \frac{1}{256} = -8$ since $2^{-8} = \frac{1}{256}$

Correct answer is: -8

7.

Solve the following equation for *x*:

$$\log_{64} x = \frac{1}{2}$$

x **=** ____

Solution:

By definition, $\log_{64} x = \frac{1}{2}$ is equivalent to $x = 64^{1/2} = 8$.

Correct answer is: 8

8.

Find
$\ln \sqrt[7]{e}$

Solution:

$\ln \sqrt[7]{e} = \ln e^{1/7}$ is the unique number c such that $e^{1/7} = e^c$; that is, $c = \frac{1}{7}$.

Hence, $\ln \sqrt[7]{e} = \frac{1}{7}$.

Correct answer is: $\frac{1}{7}$

9.

Simplify the following expression:

$$\ln e^{5x}$$

Solution:

$\ln e^{5x}$ is the unique number b for which $e^{5x} = e^b$. Clearly, this number b is $5x$.
Hence, $\ln e^{5x} = 5x$.

Correct answer is: $5x$

10.

Solve $17^{5x-1} = 29$ for x.

$x =$ _____

Solution:

$$17^{5x-1} = 29$$

$$\log_{17} 17^{5x-1} = \log_{17} 29$$

$$5x - 1 = \log_{17} 29$$
$$5x = 1 + \log_{17} 29$$
$$x = \frac{1}{5}(1 + \log_{17} 29)$$

Correct answer is: $x = \frac{1}{5}\left(1 + \log_{17} 29\right)$

11.

How quickly will money double if it is invested at an annual interest rate of 6.5% compounded continuously? Round your answer to the nearest month.

____ years and ____ months.

Solution:

$B(t) = Pe^{0.065t}$

$2 = e^{0.065t}$ and $t = \frac{\ln 2}{0.065} \approx 10.6638$ years

or 10 years 8 months.

Correct answer is: 10 years and 8 months.

12.

Differentiate the function $f(x) = x^4 \ln x$.

$f'(x) =$ ____

Solution:

Combine the product rule with the formula for the derivative of ln x to get

$$f'(x) = x^4\left(\frac{1}{x}\right) + 4x^3 \ln x = x^3 + 4x^3 \ln x$$

Correct answer is: $f'(x) = x^3 + 4x^3 \ln x$

13.

Differentiate the function $f(x) = e^{x^2 + 6}$.

$f'(x) =$ _____

Solution:

Using the chain rule with $u = x^2 + 6$, we find

$$f'(x) = e^{x^2 + 6}\left[\frac{d}{dx}(x^2 + 6)\right] = 2xe^{x^2 + 6}$$

Correct answer is: $f'(x) = 2xe^{x^2 + 6}$

14.

Differentiate the given function.

$\boldsymbol{f(x) = (8 - 20e^x)^2}$

$f'(x) =$ _____

Solution:

$$\begin{aligned} f'(x) &= 2(8 - 20e^x)\frac{d}{dx}(8 - 20e^x) \\ &= 2(8 - 20e^x)(0 - 20e^x) \\ &= -40e^x(8 - 20e^x) \end{aligned}$$

Correct answer is: $f'(x) = -40e^{x}(8 - 20e^{x})$

15.

Differentiate the given function.

$\boldsymbol{f(x) = \ln x^{17} + 84}$

$f'(x) =$ ____

Solution:

$f(x) = \ln x^{17} + 84 = 17\ln x + 84$

$$f'(x) = 17\left(\frac{1}{x}\right) + 0$$

$$= \frac{17}{x}$$

Correct answer is: $f'(x) = \frac{17}{x}$

16.

Differentiate the given function. Simplify your answer.

$\boldsymbol{f(x) = \frac{\ln 2x}{5x}}$

$f(x) = \frac{\_\_\_\_}{\_\_\_\_}$

Solution:

$$f'(x) = 5x\frac{d}{dx}(\ln 2x) - (\ln 2x)\frac{d}{dx}5x$$

$$25x^2$$

$$= \frac{1 - \ln 2x}{5x^2}$$

Correct answer is: $f(x) = \dfrac{-\ln 2x + 1}{5x^2}$

17.

Differentiate the function:

$$\boldsymbol{y = 5^x}$$

$\dfrac{dy}{dx} = $ ____

Solution:

To differentiate $y = 5^x$, we use logarithmic differentiation as follows:

$y =$	5^x	
$\ln y =$	$x(\ln 5)$	*take natural logarithms on both sides*
$\dfrac{1}{y}\dfrac{dy}{dx} =$	$\ln 5$	*differentiate both sides*
$\dfrac{dy}{dx} =$	$(\ln 5)\, y$	*multiply both sides by y*
$\dfrac{dy}{dx} =$	$(\ln 5)\, 5^x$	*substitute* $y = 5^x$

Correct answer is: $\dfrac{dy}{dx} = (\ln 5)\, 5^x$

18.

Differentiate the function.

$$y = \log_6 x$$

$$\frac{dy}{dx} = \_\_\_\_$$

Solution:

We find

$$y = \log_6 x$$

$$6^y = x \qquad \textit{definition of logarithm}$$

$$y \ln 6 = \ln x \qquad \textit{take natural logarithms on both sides}$$

$$(\ln 6)\frac{dy}{dx} = \frac{1}{x} \qquad \textit{differentiate both sides}$$

$$\frac{dy}{dx} = \frac{1}{x(\ln 6)} \qquad \textit{divide by } \ln 6$$

Correct answer is: $\frac{dy}{dx} = \frac{1}{x(\ln 6)}$
