Version A

Lecture 17 Solutions

Solutions

1.

Find the integral:

$$\int x^7\,dx$$

_____ + C

Solution:

Use the power rule with $n = 7$: $\int x^7\,dx = \frac{1}{8}x^8 + C$

Correct answer is: $\frac{1}{8}x^8 + C$

2.

Find the integral:

$$\int \frac{1}{\sqrt[3]{x}}\,dx$$

A. $\frac{3}{2}\sqrt[3]{x^2} + C$

B. $\frac{2}{3}x^3 + C$

C. $\frac{3}{2}x^3 + C$

D. $\frac{2}{3}\sqrt[3]{x^2} + C$

Solution:

Use the power rule with $n = -\frac{1}{3}$: Since $n + 1 = \frac{2}{3}$,

$$\int \frac{dx}{\sqrt[3]{x}} = \int x^{-1/3} dx = \frac{1}{2/3} x^{2/3} + C = \frac{3}{2}\sqrt[3]{x^2} + C$$

Correct answer is: A

3.

Find the following integral:

$$\int (3x^8 + 16x^3 - 2x + 3)\, dx$$

____ + C

Solution:

By using the power rule in conjunction with the sum and difference rules and the multiple rule, you get

$$\int (3x^8 + 16x^3 - 2x + 3)\, dx = 3\int x^8\, dx + 16\int x^3\, dx - 2\int x\, dx + \int 3\, dx$$

$$= 3\left(x^9 \right) + 16\left(x^4 \right) - 2\left(x^2 \right) + 3x + C$$

9 4 2

$$= \frac{1}{3}x^9 + 4x^4 - x^2 + 3x + C$$

Correct answer is: $\frac{1}{3}x^9 + 4x^4 - x^2 + 3x + C$

4.

Find the integral:

$$\int \frac{1}{x^2}\,dx$$

A. $\frac{1}{x} + C$

B. $\frac{1}{2x} + C$

C. $-\frac{1}{x} + C$

D. $-\frac{1}{2x} + C$

Solution:

$$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{1}{-1}x^{-1} + C = -\frac{1}{x} + C$$

Correct answer is: C

5.

Use the fundamental theorem of calculus to find the area of the region under the line $y = 4x + 7$ over the interval $2 \le x \le 4$.

Your Answer: ____

Solution:

The area is given by the definite integral

$$A = \int_2^4 (4x + 7)dx.$$

Since an anti derivative of $f(x) = 4x + 7$ is $F(x) = 2x^2 + 7x$,
the fundamental theorem of calculus tells us that

$$A = \int_2^4 (4x + 7)dx = 2x^2 + 7x \Big|_2^4$$

$$= [2(4)^2 + 7(4)] - [2(2)^2 + 7(2)] = 38$$

Correct answer is: 38

6.

Evaluate

$$\int_1^5 \left(\frac{1}{x} - x\right)dx.$$

Round your answer to four decimal places.

Your Answer: ____

Solution:

An antiderivative of $f(x) = \frac{1}{x} - x$ is $F(x) = \ln|x| - \frac{1}{2}x^2$,

so we have

$$\int_1^5 \left(\frac{1}{x} - x\right)dx = \left(\ln|x| - \frac{1}{2}x^2\right) \Big|_1^5$$

$$= \left[\ln 5 - \frac{1}{2}(5)^2\right] - \left[\ln 1 - \frac{1}{2}(1)^2\right]$$

$$= \ln 5 - 12 \approx -10.3906$$

Correct answer is: -10.3906
