Version A

Lecture 22 Solutions

Solutions

1.

Find $\int xe^{5x}dx$.

____ $(x -$ ____ $)e^{5x} + C$

Solution:

Although both factors x and e^{5x} are easy to integrate, only x becomes simpler when differentiated. Therefore, we choose $u = x$ and $dv = e^{5x}dx$ and find

$$u = x \qquad dv = e^{5x}dx$$

$$du = dx \qquad v = \frac{1}{5}e^{5x}$$

Substituting into the integration by parts formula, we obtain

$$\begin{aligned}\int x(e^{5x}dx) &= x\left(\frac{1}{5}e^{5x}\right) - \int\left(\frac{1}{5}e^{5x}\right)dx \\ &= \frac{1}{5}xe^{5x} - \frac{1}{5}\left(\frac{1}{5}e^{5x}\right) + C \\ &= \frac{1}{5}\left(x - \frac{1}{5}\right)e^{5x} + C\end{aligned}$$

Correct answer is: $\frac{1}{5}(x - \frac{1}{5})e^{5x} + C$

2.

Find $\int 8x^3 \ln x\, dx$.

A. $2x^4 \ln x - \frac{1}{2}x^4 + C$

B. $2x^2 \ln x - \frac{1}{2}x^2 + C$

C. $2x^4 \ln x + 2x^4 + C$

D. $2x^2 \ln x - 2x^2 + C$

Solution:

Our strategy is to express $\int 8x^3 \ln x\, dx$ as $\int u\, dv$ by choosing u and v so that $\int v\, du$ is easier to evaluate than $\int u\, dv$. This strategy suggests that we choose

$$u = \ln x \quad \text{and} \quad dv = 8x^3\, dx$$

since

$$du = \frac{1}{x}\, dx$$

is a simpler expression than ln x, while v can be obtained by the relatively easy integration

$$v = \int 8x^3\, dx = 2x^4$$

(For simplicity, we leave the "+ C" out of the calculation until the final step.)
Substituting
this choice for u and v into the integration by parts formula, we obtain

$$\int 8x^3 \ln x\, dx = \underbrace{}_{u} \underbrace{}_{dv} \quad \underbrace{}_{u} \underbrace{}_{v} \quad \underbrace{}_{v} \underbrace{}_{du}$$

$$\int (\ln x)(8x^3\,dx) = (\ln x)(2x^4) - \int (2x^4)\left(\frac{1}{x}\,dx\right)$$

$$= 2x^4 \ln x - 2\int x^3\,dx$$

$$= 2x^4 \ln x - 2\left(\frac{1}{4}x^4\right) + C$$

$$= 2x^4 \ln x - \frac{1}{2}x^4 + C$$

Correct answer is: A

3.

Use integration by parts to find the given integral.

$$\int 7x\sqrt{x-37}\,dx$$

Solution:

Both terms are easy to integrate: however, the derivative of $7x$ becomes simpler while
the derivative of $\sqrt{x-37}$ does not. So,

$u = 7x$ and $dV = (x-37)^{1/2}dx$

$du = 7dx$ $V = \frac{2}{3}(x-37)^{3/2}$ and

$$\int 7x\sqrt{x-37}\,dx = \frac{14}{3}(x-37)^{3/2}x - \int \frac{14}{3}(x-37)^{3/2}dx$$

$$= \frac{14}{3}(x-37)^{3/2}x - \frac{14}{3}\int (x-37)^{3/2}dx$$

$$= \frac{14}{3}(x-37)^{3/2}x - \frac{28}{15}(x-37)^{5/2} + C$$

Correct answer is: 14 x(x – 37) 3 – 28 (x – 37) 5 + c

2 2
3 15

4.

Use integration by parts to find the given integral.

$$\int x(x+1)^3\,dx$$

Solution:

Although both factors x and $(x + 1)^3$ are easy to integrate, only x becomes simpler when differentiated. Therefore, we choose $u = x$ and $dv = (x + 1)^3\,dx$ and find

$$u = x \qquad dv = (x+1)^3\,dx$$
$$du = dx \qquad v = \frac{1}{4}(x+1)^4$$

Substituting into the integration by parts formula, we obtain

$$\int \underbrace{x}_{u}\,\underbrace{[(x+1)^3\,dx]}_{dv} = \underbrace{x}_{u}\underbrace{\left[\frac{1}{4}(x+1)^4\right]}_{v} - \int \underbrace{\left[\frac{1}{4}(x+1)^4\right]}_{v}\,\underbrace{dx}_{du}$$
$$= \frac{1}{4}x(x+1)^4 - \frac{1}{4}\left[\frac{1}{5}(x+1)^5\right] + C$$
$$= \frac{1}{4}x(x+1)^4 - \frac{1}{20}(x+1)^5 + C$$

Correct answer is: $\frac{1}{4}x(x+1)^4 - \frac{1}{20}(x+1)^5 + c$

5.

Use integration by parts to find the given integral.

$\int$ **$28x(\ln x)^2 dx$**

Solution:

$I = \int 28x(\ln x)^2\, dx$

$u = (\ln x)^2 \qquad dv = 28x dx$

$du = \frac{2\ln x}{x} dx \qquad v = 14x^2$

$I \quad = \; 14x^2(\ln x)^2 - 28\int x\ln x dx$

Apply integration by parts to the second term with $u = \ln x$ and $dv = xdx$. Then $du = \frac{1}{x}\, dx$ and $v = \frac{1}{2}x^2$. This leads to

$= \; 14x^2(\ln x)^2 - 14x^2\ln x + \int 14x\, dx$

$= 14x^2(\ln x)^2 - 14x^2\ln x + 7x^2 + C$

Correct answer is: $\frac{28}{2}x^2(\log_e x)^2 - \frac{28}{2}x^2(\log_e x) + \frac{28}{4}x^2 + c$

6.

Find $\int 19x^2e^{2x}\, dx$.

Solution:

Since the factor e^{2x} is easy to integrate and $19x^2$ is simplified by differentiation, we choose

$$u = 19x^2 \qquad dv = e^{2x}\, dx$$

so that

$$du = 38x\,dx \qquad v = \int e^{2x}\,dx = \frac{1}{2}e^{2x}$$

Integrating by parts, we get

$$\int 19x^2 e^{2x}\,dx = 19x^2\left(\frac{1}{2}e^{2x}\right) - \int\left(\frac{1}{2}e^{2x}\right)(38x\,dx)$$

$$= \frac{19}{2}x^2e^{2x} - \int 19xe^{2x}\,dx$$

The integral $\int 19xe^{2x}$ that remains can also be obtained using integration by parts.
We found that

$$\int 19xe^{2x}\,dx = \frac{19}{2}\left(x - \frac{1}{2}\right)e^{2x} + C$$

Thus,

$$\int 19x^2e^{2x}\,dx = \frac{19}{2}x^2e^{2x} - \int 19xe^{2x}\,dx$$

$$= \frac{19}{2}x^2e^{2x} - \left[\frac{19}{2}\left(x - \frac{1}{2}\right)e^{2x}\right] + C$$

$$= \frac{19}{4}(2x^2 - 2x + 1)e^{2x} + C$$

Correct answer is: $\frac{19}{4}(2x^2 - 2x + 1)e^{2x} + c$

7.

Use integration by parts to find the given integral.

$$\int_1^3 (11t - 42)e^{6-t}\,dt$$

Solution:

$$I = \int (11t - 42)e^{6-t}dt$$

$$u = 11t - 42 \qquad dv = e^{6-t}dt$$

$$du = 11dt \qquad v = -e^{6-t}$$

$$\begin{aligned} I &= -(11t-42)e^{6-t} - \int (-11e^{6-t})dt \\ &= -11te^{6-t} + 31e^{6-t} + C \\ \int_1^3 (11t-42)e^{6-t}dt &= \left[-11te^{6-t} + 31e^{6-t}\right]\Big|_1^3 \\ &= -2e^3 - 20e^5 \end{aligned}$$

Correct answer is: $-2e^3 - 20e^5$
