Version A

Lecture 24 Solutions

Solutions

1.

Find the partial derivative f_x if $f(x, y) = 2x^3 + 7xy^4 + \frac{3y}{4x}$.

$f_x(x, y) =$ ____

Solution:

To simplify the computation, begin by rewriting the function as

$$f(x, y) = 2x^3 + 7xy^4 + \frac{3}{4}yx^{-1}$$

To compute f_x, think of f as a function of x and differentiate the sum term by term, treating y as a constant to get

$$f_x(x, y) = 2(3)x^2 + 7(1)y^4 + \frac{3}{4}y(-x^{-2})$$

$$= 6x^2 + 7y^4 - \frac{3y}{4x^2}$$

Correct answer is: $f_x(x, y) = 6x^2 + 7y^4 - \frac{3y}{4x^2}$

2.

Find the partial derivative ∂z if $z = (x^2 + xy + y)^6$.

∂y

A. $(x^2 + xy + x)^6(x + 1)$

B. $6(x^2 + xy + x)^5(2x + y)$

C. $6(x^2 + xy + x)^5$

D. $6(x^2 + xy + x)^5(x + 1)$

Solution:

Holding x fixed and using the chain rule to differentiate z with respect to y, you get

$$\frac{\partial z}{\partial y} = 6(x^2 + xy + x)^5 \frac{\partial}{\partial y}(x^2 + xy + y)$$

$$= 6(x^2 + xy + x)^5(x + 1)$$

Correct answer is: D

3.

Find the partial derivative f_x if $f(x, y) = 4xe^{-7xy}$.

$f_x(x, y) =$ ____

Solution:

From the product rule,

$$f_x(x, y) = 4x(-7ye^{-7xy}) + 4e^{-7xy}$$

$$= 4(-7xy + 1)e^{-7xy}$$

Correct answer is: $f_x(x, y) = 4(- 7xy + 1)e^{-7xy}$

4.

Evaluate the partial derivative $f_x(x, y)$ at the point $P_0(0, 0)$.

$$f(x, y) = 8xe^{-2y} + 4ye^{-3x} + 5xy^5$$

$f_x(0, 0) =$ ____

Solution:

To compute f_x, think of f as a function of x and differentiate the sum term by term, treating y as a constant to get

$$\begin{aligned} f_x(x, y) &= 8(1)e^{-2y} + 4y(-3e^{-3x}) + 5(1)y^5 \\ &= 8e^{-2y} - 12ye^{-3x} + 5y^5 \end{aligned}$$

Substituting x = 0 and y = 0 into the formula for $f_x(x, y)$, we get

$$\begin{aligned} f_x(0, 0) &= 8e^0 - 12(0)e^0 + 5(0)^5 \\ &= 8 - 0 + 0 = 8 \end{aligned}$$

Correct answer is: 8

5.

Compute all first-order partial derivative of the given function.

$$f(x, y) = \frac{7x + 5y}{9y - 7x}$$

$f_x = \dfrac{\_\_\_\_}{(9y - 7x)^2}$, $f_y = \dfrac{\_\_\_\_}{(9y - 7x)^2}$

Solution:

$$f_x = \frac{(9y - 7x)(7) - (7x + 5y)(-7)}{(9y - 7x)^2}$$

$$= \frac{98y}{(9y - 7x)^2}$$

$$f_y = \frac{(9y - 7x)(5) - (7x + 5y)(9)}{(9y - 7x)^2}$$

$$= \frac{-98x}{(9y - 7x)^2}$$

Correct answer is: $f_x = \dfrac{98y}{(9y - 7x)^2}$, $f_y = \dfrac{-98x}{(9y - 7x)^2}$

6.

Compute the second-order partial derivative f_{xx} of the function $f(x, y) = xy^6 + 7xy^2 + 2x + 1$

A. $6y^5 - 14y$

B. 0

C. $6y^5 + y$

D. $30xy^4 + 14x$

Solution:

Since

$$f_x = y^6 + 7y^2 + 2$$

it follows that

$$f_{xx} = 0$$

Correct answer is: B

7.

Find the second partial derivative f_{xy} if $f(x, y) = 6x^7y^7 + 6xy$.

$f_{xy} =$ ____

Solution:

Since

$f_x = 42x^6y^7 + 6y$

if follows that

$f_{xy} = 294x^6y^6 + 6$

$= 6(49x^6y^6 + 1)$

Correct answer is: $f_{xy} = 6(49x^6y^6 + 1)$

8.

Find the second partial derivative f_{yy} if $f(x, y) = 8x^3y^5 + 4xy$.

$f_{yy} =$ ____

Solution:

Since

$f_y = 40x^3y^4 + 4x$

we have

$f_{yy} = 160x^3y^3$

Correct answer is: $f_{yy} = 160x^3y^3$

9.

Find the second partials.

$f(u, v) = 4\ln(u^2 + v^2)$

f_{uv}= ____ , f_{vv}= ____

Solution:

$$f_u = \frac{8u}{u^2 + v^2}$$

$$f_{uu} = 8\left(\frac{u^2 + v^2 - 2u^2}{(u^2 + v^2)^2}\right) = \frac{8(v^2 - u^2)}{(u^2 + v^2)^2}$$

Rewrite f(u, v) and what follows above with u and v interchanged.

Then $f_{vv} = \dfrac{8(u^2 - v^2)}{(u^2 + v^2)^2}$ without performing any additional computations.

$$f_{uv} = f_{uv} = 8(2u)(-1)(u^2 + v^2)^{-2}(2v)$$

$$= -\frac{32uv}{(u^2 + v^2)^2}$$

Correct answer is: $f_{uv} = \dfrac{-32uv}{(u^2 + v^2)^2}$, $f_{vv} = \dfrac{-8u^2 + 8v^2}{(u^2 + v^2)^2}$
