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$\boxed{4.1 \ \#1, 9, 13, 41}$

1. $4^{x} = (2^{2})^{x} = \boxed{2^{2x}}$,

$(\sqrt{3})^{x} = 3^{x/2} = \boxed{3^{1/2 \cdot x}}$,

$\left(\frac{1}{9}\right)^{x} = \left(\frac{1}{3^{2}}\right)^{x} = (3^{-2})^{x} = \boxed{3^{-2x}}$

9. $\frac{3^{4x}}{3^{2x}} = 3^{4x-2x} = \boxed{3^{2x}}$,

$\frac{2^{5x+1}}{2 \cdot 2^{-x}} = \frac{2^{5x+1}}{2^{1-x}} = 2^{5x+1-(1-x)} = \boxed{2^{6x}}$,

$\frac{9^{-x}}{27^{-x/3}} = \frac{(3^{2})^{-x}}{(3^{3})^{-x/3}} = \frac{3^{-2x}}{3^{-x}} = 3^{-2x-(-x)} = \boxed{3^{-x}}$

13. $(2^{-3x} \cdot 2^{-2x})^{2/5} = (2^{-3x+(-2x)})^{2/5} = (2^{-5x})^{2/5} = \boxed{2^{-2x}}$

$(9^{1/2} \cdot 9^{4})^{x/9} = \left[(3^{2})^{1/2}(3^{2})^{4}\right]^{x/9} = (3^{2/2} \cdot 3^{8})^{x/9}$

$= (3 \cdot 3^{8})^{x/9} = (3^{1+8})^{x/9} = 3^{\frac{9x}{9}} = 3^{x} = \boxed{3^{x}}$

(2)

41. $3^{x/2} + 3^{-x/2} = 3^{-x/2}(\quad)$.

$$3^{x/2} + 3^{-x/2} = \frac{3^{-x/2}}{3^{-x/2}} \cdot 3^{x/2} + 3^{-x/2} = 3^{-x/2} \cdot \frac{3^{x/2}}{3^{-x/2}} + 3^{-x/2}$$

$$= 3^{-x/2}\left[\frac{3^{x/2}}{3^{-x/2}} + 1\right]$$

$$= 3^{-x/2}\left[3^{x/2-(-x/2)} + 1\right]$$

$$= \boxed{3^{-x/2}\left(3^{x}+1\right)}.$$

$\boxed{4.2 \ \# \ 33, 34, 35, 42}$

Note: $\frac{d}{dx}(e^x) = e^x$,
$\frac{d}{dx}(b^x) = b^x \log b$.

$\frac{d}{dx}(b^x) = mb^x$, where $m = \frac{d}{dx}(b^x)\Big|_{x=0}$.

m is also given to be $m = \lim_{h\to 0} \frac{b^h - 1}{h}$

33. Differentiate: $\frac{e^x}{1+e^x}$.

QUOTIENT RULE

$$\frac{d}{dx}\left(\frac{e^x}{1+e^x}\right) = \frac{(1+e^x)e^x - e^x(e^x)}{(1+e^x)^2}$$

$$= \frac{e^x + \cancel{e^{2x}} - \cancel{e^{2x}}}{(1+e^x)^2} = \boxed{\frac{e^x}{(1+e^x)^2}}$$

34. $\frac{d}{dx}\left((1+x^2)e^x\right) \overset{\text{PRODUCT RULE}}{=} (1+x^2)e^x + 2x(e^x)$

$= e^x + x^2e^x + 2x\cdot e^x$

$= e^x(1+2x+x^2)$

$= \boxed{e^x(1+x)^2}$

35. $\frac{d}{dx}\left((1+5e^x)^4\right) \overset{\text{chain rule}}{=} \boxed{4(1+5e^x)^3(5e^x)}$

42. Find the equation of the tangent line to the curve $y = \frac{e^x}{x+e^x}$ at $(0,1)$.

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x}{x+e^x}\right) \overset{\text{QUOTIENT RULE}}{=} \frac{(x+e^x)e^x - e^x(1+e^x)}{(x+e^x)^2}$

$\left.\frac{dy}{dx}\right|_{x=0} = \frac{(0+e^0)e^0 - e^0(1+e^0)}{(0+e^0)^2} = \frac{(1)1-1(1+1)}{(1)^2}$

$= \frac{1-2}{1} = -1$

So $\boxed{y-1=-1(x-0)}$.

④

$\boxed{4.3 \ \# \ 3, 12, 13, 16, 18, 21, 32, 34, 44, 46}$

3. $f(x) = 1 + 4x + e^{-2x}$

$$\boxed{f'(x) = 4 + (e^{-2x})(-2)}$$

12. $f(t) = 5e^{3t-7}$

$$\boxed{f'(t) = (5e^{3t-7})(3)}$$

$f(t) = Ae^{g(t)}$

$f'(t) = Ae^{g(t)}(g'(t))$

13. $y = e^{x^2-5x+4}$

$$\boxed{y'(x) = (e^{x^2-5x+4})(2x-5)}$$

16. $f(t) = \dfrac{1}{e^{3t+1}}$

METHOD 1:

$$f'(t) = (e^{-(3t+1)})' = (e^{-3t-1})' = \boxed{(e^{-3t-1})(-3)}$$

METHOD 2:

$$f'(t) = \frac{d}{dt}\left(\frac{1}{e^{3t+1}}\right) \overset{\text{QUOTIENT RULE}}{=} \frac{(e^{3t+1})(0) - 1(e^{3t+1})(3)}{e^{2(3t+1)}}$$

$$= -\frac{\cancel{(e^{3t+1})}(3)}{\cancel{e^{3t+1}} \cdot e^{3t+1}} = -\frac{3}{e^{3t+1}} = \boxed{-3e^{-(3t+1)}}$$

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18. $f(t) = e^{t}\left(e^{2t} - e^{-2t}\right)$

$f(t) = e^{3t} - e^{t-2t} = e^{3t} - e^{-t}$.

So $\boxed{f'(t) = e^{3t}(3) - e^{-t}(-1)}$

21. $f(x) = x^3 e^{2x}$

PRODUCT RULE

$\boxed{f'(x) = x^3\left(e^{2x}\right)(2) + 3x^2\left(e^{2x}\right)}$

32. $(2x-5)e^{3x-1}$

Product Rule

$\frac{d}{dx}\left((2x-5)e^{3x-1}\right) = (2x-5)\left(e^{3x-1}\right)(3) + 2\left(e^{3x-1}\right)$

Find local max / local min.

$\frac{d}{dx}\left((2x-5)e^{3x-1}\right) = 0$.

$\left[3(2x-5) + 2\right]\left(e^{3x-1}\right) = 0$

Since $e^{3x-1} \neq 0$ for all x, $3(2x-5)+2=0$.

So $6x - 15 + 2 = 6x - 13 = 0$

$\boxed{x = \frac{13}{6}}$

Is it local max or local min?

$\frac{d^2}{dx^2}\left((2x-5)e^{3x-1}\right) = \frac{d}{dx}\left((6x-15+2)\left(e^{3x-1}\right)\right)$

$= \frac{d}{dx}\left((6x-13)\left(e^{3x-1}\right)\right) \longrightarrow$

⑥

$$\underset{\substack{\uparrow \\ \text{product} \\ \text{rule}}}{=} (6x-13)(e^{3x-1})(3) + 6(e^{3x-1})$$

So $\displaystyle \frac{d^2}{dx^2}\left((2x-5)e^{3x-1}\right)\Big|_{x=\frac{13}{6}} = (6x-13)(e^{3x-1})(3) + 6(e^{3x-1})\Big|_{x=\frac{13}{6}}$

$$= (13-13)\,e^{3(\frac{13}{6})-1}(3) + 6\left(e^{\frac{13}{2}-1}\right)$$

$$= 6\,e^{\frac{13}{2}-1} > 0 \quad (\text{concave up})$$

$$\boxed{\text{So } x = \tfrac{13}{6} \text{ is a relative minimum.}}$$

34. $v(t) = 2000\,e^{-.35t}$

$v'(t) = 2000\left(e^{-.35t}\right)(-.35)$

$$\boxed{v'(3) = (-.35)(2000)\,e^{-.35(3)}}$$

44. Use the following fact:

Let C, k be any constants and let $y = Ce^{kx}$.
Then y satisfies the equation $y' = ky$.

Also, if $y = f(x)$ satisfies $y' = ky$, then
y is an exponential function of the form
$y = Ce^{kx}$, where C is a constant.

44. (Cont.) So $y' = 3y$ means $k = 3$.

So $y = Ce^{kx} = Ce^{3x} = f(x)$.

Since $f(0) = \frac{1}{2}$, $f(0) = Ce^{3(0)} = \frac{1}{2}$

So $C \cdot e^0 = \frac{1}{2}$

$\Rightarrow \quad C \cdot 1 = \boxed{C = \frac{1}{2}}$

So $\boxed{y = f(x) = \frac{1}{2} e^{3x}}$

46. Let $f(x)$ be a function with the property that $f'(x) = \frac{1}{x}$. Let $g(x) = f(e^x)$, and compute $g'(x)$.

$$g'(x) = \left(f(e^x)\right)' \overset{\text{CHAIN RULE}}{=} f'(e^x) \cdot e^x$$

$$= \frac{1}{e^x} \cdot e^x = 1.$$

$\boxed{\text{So } g'(x) = 1.}$