Mee Seong Im MATH234 - Exam 3 Version 1 November 17, 2007

1. An investment of \$500 earning interest compounded continuously doubles in 7 years. Answer the questions below. Show your work.

a. What is the annual interest rate, r, for the investment?
ANSWER:

$$A(t) = P_0 e^{rt} \quad (1)$$
$$1000 = 500 e^{r(7)} \quad (2)$$
$$2 = e^{7r} \quad (3)$$
$$\ln 2 = 7r \quad (4)$$
$$\frac{\ln 2}{7} = r \quad (5)$$

b. How long does it take the original investment to triple in size?
ANSWER:

$$1500 = 500 e^{rt} \quad (6)$$
$$3 = e^{rt} \quad (7)$$
$$\ln 3 = rt \quad (8)$$
$$\frac{\ln 3}{r} = t \quad (9)$$
$$\frac{\ln 3}{\left(\frac{\ln 2}{7}\right)} = t \quad (10)$$
$$7\frac{\ln 3}{\ln 2} = t \quad (11)$$

2. The demand function for a certain commodity is

$$q = 600e^{-.2p}.$$

a. The current price is \$10. Is demand elastic or inelastic at $p = 10$? Show your work.
ANSWER: From the demand function (Chapter 5.3), we have

$$E(p) = \frac{-pf'(p)}{f(p)} = \frac{-p\,(-.2)600e^{-.2p}}{600e^{-.2p}} = .2p.$$

So plug-in to this function when $p = 10$ to obtain $E(10) = .2(10) = 2$.
Since $E(10) = 2 > 1$, the demand is elastic at $p = 10$.

b. If price is lowered slightly, will revenue increase or decrease?
ANSWER: Since the demand is elastic, lowering the price will INCREASE revenue.

3. Compute the following indefinite integrals:

a. $\int \left(\sqrt[5]{t} - \frac{7}{t} + t^{-7/3} \right) dt$

ANSWER:

$$= \frac{5}{6} t^{6/5} - 7 \ln |t| - \frac{3}{4} t^{-4/3} + C$$

b. $\int \left(e^{4x} + \frac{1}{6} x^3 + e^5 \right) dx$

ANSWER:

$$= \frac{1}{4} e^{4x} + \frac{1}{6} \cdot \frac{1}{4} x^4 + e^5 x + C$$

4. Answer the questions below. Show your work.

a. Write down the definite integral or sum of definite integrals that gives the area of the shaded region in the graph below (see your exam for the graph):

ANSWER: The area is equal to

$$\int_0^2 (g(x) - h(x))\, dx + \int_2^5 (f(x) - h(x)) dx + \int_5^8 (h(x) - f(x)) dx$$

b. Evaluate the following definite integral:

$$\int_3^4 e^{3x} - 2\, dx$$

ANSWER:

$$\int_3^4 e^{3x} - 2dx = \left(\frac{1}{3} e^{3x} - 2x \right) |_3^4 \tag{12}$$

$$= \left(\frac{1}{3} e^{12} - 8 \right) - \left(\frac{1}{3} e^9 - 6 \right) \tag{13}$$

$$= \frac{1}{3} e^{12} - \frac{1}{3} e^9 - 2 \tag{14}$$

5. What is the consumers' surplus for the demand curve $p = 8 - \frac{x}{6}$ at the sales level $x = 12$? Show your work.

ANSWER:

$$\int_0^A (p(x) - p(A)) dx = \int_0^{12} \left(\left(8 - \frac{x}{6} \right) - \left(8 - \frac{12}{6} \right) \right) dx \tag{15}$$

$$= \int_0^{12} \left(8 - \frac{x}{6} - 6 \right) dx \tag{16}$$

$$= \int_0^{12} \left(2 - \frac{x}{6} \right) dx \tag{17}$$

$$= \left(2x - \frac{x^2}{12} \right) |_0^{12} \tag{18}$$

$$= (24 - 12) - 0 = 12 \tag{19}$$

6. Evaluate the following integrals using the specified method. Show your work.

a. Substitution:

$$\int \left(x^2 + 2e^{2x}\right) \sqrt{x^3 + 3e^{2x}} dx$$

ANSWER:

$$u = x^3 + 3e^{2x} \tag{20}$$
$$du = 3x^2 + 6e^{2x} dx \tag{21}$$
$$\frac{1}{3} du = \left(x^2 + 2e^{2x}\right) dx \tag{22}$$

So

$$\int \left(x^2 + 2e^{2x}\right) \sqrt{x^3 + 3e^{2x}} dx = \int \sqrt{u} \left(\frac{1}{3} du\right) \tag{23}$$
$$= \frac{1}{3} \int u^{1/2} du \tag{24}$$
$$= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C \tag{25}$$
$$= \frac{2}{9} \left(x^3 + 3e^{2x}\right)^{3/2} + C \tag{26}$$

b. Integration by Parts:

$$\int xe^{-x/2} dx$$

ANSWER:

$u = x$	$v = -2e^{-x/2}$
$du = dx$	$dv = e^{-x/2} dx$

$$\int udv = uv - \int vdu \tag{27}$$
$$\int xe^{-x/2} dx = x(-2e^{-x/2}) - \left(-2e^{-x/2}\right) dx \tag{28}$$
$$= -2xe^{-x/2} + 2 \int e^{-x/2} dx \tag{29}$$
$$= -2xe^{-x/2} + 2 \left(-2e^{-x/2}\right) + C \tag{30}$$
$$= -2xe^{-x/2} - 4e^{-x/2} + C \tag{31}$$

7. Evaluate the following integral. Points will be given only for significant progress. Show your work.

$$\int (2x + e^x) \ln \left(x^2 + e^x + 1\right)^{3/2} dx$$

ANSWER: First use substition. Then apply integration by parts.
So let $u = x^2 + e^x + 1$. Then $du = (2x + e^x)\,dx$. Then

$$\int (2x + e^x) \ln \left(x^2 + e^x + 1\right)^{3/2} dx = \int \ln(u)^{3/2} du \tag{32}$$

$$= \int \frac{3}{2} \ln u du \tag{33}$$

$$= \int \frac{3}{2} \ln y dy, \quad \text{where } u = y \tag{34}$$

Now apply integration by parts.

$u = \ln y$	$v = \frac{3}{2}y$
$du = \frac{1}{y}dy$	$dv = \frac{3}{2}dy$

So

$$\int \frac{3}{2} \ln y dy = \ln y \left(\frac{3}{2}y\right) - \int \frac{3}{2}y \left(\frac{1}{y}dy\right) \tag{35}$$

$$= \frac{3}{2}y \ln y - \int \frac{3}{2}dy \tag{36}$$

$$= \frac{3}{2}y \ln y - \frac{3}{2}y + C \tag{37}$$

$$= \frac{3}{2}u \ln u - \frac{3}{2}u + C \quad \text{since } u = y \tag{38}$$

$$= \frac{3}{2} \left(x^2 + e^x + 1\right) \ln \left(x^2 + e^x + 1\right) - \frac{3}{2} \left(x^2 + e^x + 1\right) + C \tag{39}$$