Problem 3.24 [page 73]. Let m be a natural number. Find the flaw in the statement below. Explain why the statement is not valid, and change one symbol to correct it.

If T is a set of natural numbers such that 1) $m \in T$ and 2) $n \in T$ implies $n+1 \in T$, then $T = \{n \in \mathbb{N} : n \geq m\}$.

Solution. Using set notations, T should be written as the following:

$$T = \{n \in \mathbb{N} : 1.\ m \in T \text{ and} \tag{1}$$
$$2.\ n \in T \Rightarrow n+1 \in T\}. \tag{2}$$

So let S be defined as $S := \{n \in \mathbb{N} : n \geq m\}$, as in the book.

Check : Is it true that $S \subseteq T$?

Let n be in S. Then n is greater than or equal to m. Since m is in T, we have $m+1$, $m+2$, $m+3$,... are all in T. So since n satisfies the conditions to be in T, n is in T. So T contains S, i.e., $T \supseteq S$.

Check : Is it true that $T \subseteq S$?

Now let n be in T. If n is greater than or equal to m, then n is in S. If n is less than m, then n is not in S.

Thus $T \nsubseteq S$ because it is possible that T may be a proper set containing S.

So the mistake is the equal sign after the word *then*, and a corrected version is:

If T is a set of natural numbers such that 1) $m \in T$ and 2) $n \in T$ implies $n+1 \in T$, then $T \supseteq \{n \in \mathbb{N} : n \geq m\}$.