Mee Seong Im   Introduction to Logic and Proofs   Tuesday June 24, 2008

Problem 3.47 [page 74]. Prove that $5^n + 5 < 5^{n+1}$ for all $n \in \mathbb{N}$.

[Everyone did this problem in so many different ways but not many of you did this using proof by induction.]

Proof. Base case: show that the above inequality holds when $n = 1$. So $5^n + 5 = 5^1 + 5 = 10 < 5^{n+1} = 5^{1+1} = 5^2 = 25$. [Good, it works!]

Now assume that the above inequality holds when $n = k$:

$$5^k + 5 < 5^{k+1}.$$

Prove the inequality for $n = k + 1$:

$$
\begin{aligned}
5^{(k+1)} + 5 = 5\left(5^k + 1\right) &< 5\left(5^k + 5\right) && (1)\\
&< 5\left(5^{k+1}\right) \text{ by our assumption} && (2)\\
&= 5^{1+k+1} && (3)\\
&= 5^{(k+1)+1} && (4)
\end{aligned}
$$

Thus $5^n + 5 < 5^{n+1}$ holds for all $n \in \mathbb{N}$.