Problem 5.18 [page 119]. Count the sets of 6 cards from a standard deck of 52 cards that have at least one card in every suit.

There were many different solutions to 5.18.

If you wrote your solution as

$$\binom{13}{1}\binom{13}{1}\binom{13}{1}\binom{13}{1}\binom{48}{1}\binom{47}{1},$$

then your solution counts certain hands more than once. For example, let s =spades, h = hearts, d = diamonds and c = clubs. The hand: 2s, 3h, 4d, 2c, 3s, 8d and the hand 3s, 3h, 8d, 2c, 2s, 4d are the same, but will be counted twice in your count. We can avoid that by dividing the hands into two classes: those in which one suit will occur 3 times and the other suits once each and those hands in which two of the suits occur twice and the other two suits occur once each.

Thus in the first case the answer is (4 choose 1) (this chooses the suit that will occur three times) (13 choose 3) (the number of ways to choose the cards of that suit) (13 choose 1) (13 choose 1) (13 choose 1).

In the second case the answer is (4 choose 2) (the number of ways to choose the two suits that will occur twice) (13 choose 2) (13 choose 2) (13 choose 1) (13 choose 1). The final answer is then the sum of these two:

$$\binom{4}{1}\binom{13}{3}\binom{13}{1}\binom{13}{1}\binom{13}{1}+\binom{4}{2}\binom{13}{2}\binom{13}{2}\binom{13}{1}\binom{13}{1}.$$

Solution by Dr. Paul Weichsel.