Problem 5.29 [page 119]. Count the solutions in positive integers $x_1, ..., x_k$ to $x_1 + x_2 + ... + x_k = n$.

Solution. The formula in the text is the number of ways to write n as a sum of k NONNEGATIVE integers. The problem here asks for the ways to write n as a sum of k POSITIVE integers. If you remember the proof of the theorem that involves circles and bars, and translate this to a sum of integers, we can't have two bars next to each other because that represents a zero. In other words think of a list of n circles (each circle represents the integer 1) and then count how many ways there are to place k-1 bars between the circles so that no two bars are adjacent. If we number the circles with 1, 2, 3, ..., n, then the spaces in between them can be represented by the ordered pairs (1, 2), (2, 3), and so on. Then we can ask how many ways can we choose k-1 of these ordered pairs. Since there are n-1 such pairs altogether, there are (n-1 choose k-1) such pairs and each one represents a different way (the order counts) of writing n as a sum of k positive integers.

Solution by Dr. Paul Weichsel.