

1. Prove that if $m|a$ and $m|b$, then $m|\gcd(a, b)$
2. Show if $a|n$ and $b|n$, then $ab|n \cdot \gcd(a, b)$.
3. Prove $\text{lcm}(a, b) = \frac{ab}{\gcd(a, b)}$.

1. **Proof.** There are some integers s and t so that $\gcd(a, b) = as + bt$. Since $m|a$ and $m|b$, $m|as + bt$. So $m|\gcd(a, b)$.

2. **Proof.** Since $a|n$, there is some integer k so that $n = ka$. Since $b|n$, there is some $l \in \mathbb{Z}$ so that $n = lb$. So

$$n \cdot \gcd(a, b) = n(as + bt) = nas + nbt = (lb)as + (ka)bt = (ls)ab + (kt)ab.$$

Since ab divides the right hand side, ab divides $n \cdot \gcd(a, b)$ as well.

3. **Proof.** Take $n = \text{lcm}(a, b)$. Then by Part 2, $ab|\text{lcm}(a, b) \cdot \gcd(a, b)$. So $ab \leq \text{lcm}(a, b) \cdot \gcd(a, b)$, i.e., $\frac{ab}{\gcd(a, b)} \leq \text{lcm}(a, b)$.

On the other hand, there exist p and q so that $a = p \cdot \gcd(a, b)$ and $b = q \cdot \gcd(a, b)$. So $ab = pq \gcd(a, b)$ and $ab = aq \gcd(a, b)$. So $ab/\gcd(a, b) (= pq = aq)$ is an integer multiple of a and b . Since $n = \text{lcm}(a, b)$, $n|\frac{ab}{\gcd(a, b)}$.

$$\text{So } n \leq \frac{ab}{\gcd(a, b)}, \text{ i.e., } \text{lcm}(a, b) \leq \frac{ab}{\gcd(a, b)}.$$

$$\text{Hence } \text{lcm}(a, b) = \frac{ab}{\gcd(a, b)}.$$