

1. Practice problems

Questions: True or false? If false give counterexample

- (1) Linear independent vectors are orthogonal.
- (2) Every matrix $n \times n$ matrix a can be written as

$$a = LB$$

where B comes from a basis transform and L is lower triangular.

- (3) Let $v_1, \dots, v_m \in \mathbb{R}^k$. The Gram-Schmidt procedure gives m orthogonal vectors b_1, \dots, b_m .

Problems:

- (1) Let $C = \{(x, y) : x^2 + 2y^2 = 1\}$. Find curvature and frame at $(1/\sqrt{2}, 1/2)$.
- (2) Find area and angle for $v_1(x) = \cos(x)$ and $v_2(x) = \sin(2x)$ in $C[-\pi, \pi]$ with

$$(f, g) = \int_{-\pi}^{\pi} f(x)g(x)dx.$$

- (3) Given the two lines

$$L_1 = \{(x, y, z) : x + y + z = 1, x + 2y + z = 1\}$$

and

$$L_2 = \{(x, y, z) : x + 2y + 3z = 1, 3x + y + 2z = 1\}.$$

Is there an intersection?

- (4) What initial velocity is needed if you start a projectile at $(0, 0, 0)$ and want to shoot it to $(10, 10, 10)$?
- (5) Find the moving frame and curvature for

$$C = \{(x \cos(x), x \sin(x), x^2) : 0 \leq x \leq 2\pi\}$$

Make a picture.

- (6) Let $H = \{(x, y, z, w) : x + y + z + w = 5\}$. Find a parametrization.