

Math 406-History of Calculus- Homework 1

Due date: September 9

- (1) We say that m_1 and m_2 are commensurable if there exists natural numbers such that $km_1 = lm_2$. Show that the length of the diagonal of square with length 1 is not commensurable with the side-length 1. What if the side length is 2? What does that say about stretching squares?
- (2) Let m_1, m_2 be objects of the same type and s_1, s_2 be objects of the same type. Assume that

$$km_1 > lm_2 \quad \text{if and only if} \quad ks_2 > ls_2$$

holds for all natural numbers k, l . Show that according to Eudoxus definition we have $(m_1 : m_2) = (s_1 : s_2)$.

- (3) Use you knowledge from real analysis (an ε - δ definition) to show that for real numbers Eudoxus definition and

$$\frac{m_1}{m_2} = \frac{s_1}{s_2}$$

is equivalent.

- (4) Read about the double reduction ad absurdum argument and describe in your words.
- (5) Let us assume that m_1, m_2 are of the same type and s_1, s_2 are of the same type. Let p_n, q_n and v_n, w_n such that

- i) p_n, q_n are of the same type as m_1 and m_2 , $p_n \leq m_1$, $q_n \leq m_2$ and

$$m_1 - p_n < 2^{-n}m_1 \quad \text{and} \quad m_2 - q_n < 2^{-n}m_2;$$

- ii) v_n, w_n are of the same type as s_1 and s_2 , $v_n \leq s_1$, $w_n \leq s_2$ and

$$s_1 - v_n < 2^{-n}s_1 \quad \text{and} \quad s_2 - w_n < 2^{-n}s_2;$$

- ii) $(p_n : q_n) = (v_n : w_n)$.

Show that $(m_1 : m_2) = (s_1 : s_2)$.