Practice problems for exam 2

1. Recall Fermat's solution for calculating

$$\int_0^a x^{p/q} dx$$

for $p, q \in \mathbb{N}$. Let $0 < r < 1$ and $x_n = ar^n$. Show that

$$\lim_{r\to 1} \sum_{n=0}^{\infty} (f(x_n) - f(x_{n+1}))(x_n - x_{n+1}) \;=\; 0\,.$$

Here $f(x) = x^{p/q}$. By the way this shows that

$$\int_0^a x^{p/q} dx \;=\; \lim_{r\to 1} \sum_{n=0}^{\infty} f(x_n)(x_n - x_{n+1})\,.$$

2. We want to use the Greek method of exhaustion with finitely many rectangles to calculate

$$\int_0^1 x^{3/2} dx\,.$$

For a given $n \in \mathbb{N}$, we use the points $x_k = (\frac{k}{n})^2$ (why).

a) Show that $\lim_n \sum_{k=1}^n f(x_k)(x_{k+1} - x_k) = \frac{2}{5}$.

b) Show that $\lim_n \sum_{k=1}^n (f(x_{k+1}) - f(x_k))(x_{k+1} - x_k) = 0$.

c) Conclude that

$$\int_0^1 x^{3/2} dx \;=\; \frac{2}{5}\,.$$

3. Explain the main principles in Cavalieri's calculus using the notation

$$a^3 \;=\; \int_0^a a^2 dx \;=\; \int_0^a (x + (a-x))^2 dx\,.$$

4. Read page 124/125 solve exercise 3 on page 125.

5. Exercise 8 on page 130.