

Homework 6-Math444

Name:

Due date: Wednesday, March 5.

In the following problems try to use the definition first.

- (1) Show that union of two closed is closed.
- (2) Show that union of two compact sets is compact.
- (3) Let V_n be all open sets. Show that

$$V = \bigcup_n V_n = \{x : \exists_n x \in V_n\}$$

is open.

- (4) Let $f : (a, b) \rightarrow \mathbb{R}$ be a continuous function and $O \subset \mathbb{R}$ be open. Show that

$$f^{-1}(O) = \{x : f(x) \in O\}$$

is open.

- (5) Let $f_1, f_2 : \mathbb{R} \rightarrow \mathbb{R}$ be continuous functions show that

$$S = \{x : f_1(x) \geq 0 \text{ and } f_2(x) \geq 0\}$$

is closed. Apply this to show that

$$\{x : (x - 3)^2 \geq 8 \text{ and } (x - 7)^{800} \leq 18000\}$$

is closed.