

Real Analysis-Homework 6

Due date: Monday, October 18

(1) Let us denote by $p_k(t) = t^k$, $k \in \mathbb{N}_0$ the polynomials. Show that

$$\{a_0p_0 + a_1p_1 + a_2p_2 : a_0, a_1, a_2 \in \mathbb{R}\}$$

is a closed subset of $C[0, 1]$. (Hint: Show that for $R > 0$ the set

$$\{(a_0, a_1, a_2) : \sup_{0 \leq t \leq 1} |a_0 + ta_1 + t^2a_2| \leq R\}$$

is closed and bounded.)

(2) Let $\phi : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by

$$\phi(x_1, x_2) = \left(2x_1, \frac{1}{2}x_2\right).$$

Show that for every Lebesgue measurable subset E of \mathbb{R}^2 .

$$m(\phi^{-1}(E)) = m(E).$$

- (3) (a) Use an enumeration of the rational numbers in $[0, 1]$ and construct an open set of measure $< \frac{1}{n}$ which contains the rational numbers.
(b) Construct a meager subset of $[0, 1]$ of measure 1.