Real Analysis-Homework 7

Due date: Monday, October 25

(1) (25P) Let $P \subset [0,1]$ be the non measurable set constructed in class (as set of representatives of $x \sim y$ iff $x - y \in \mathbb{Q}$). Show that for every measurable subset $E \subset P$ we have $m(E) = 0$. (Royden: 3.15)

(2) (25P) Show that every set with $0 < m^*(A) < \infty$ contains a non measurable set (Royden 3.16).

(3) (25P) Show Proposition 24 on page 73 (Royden): Let E be a measurable set of finite measure and (f_n) a sequence of measurable functions which converges to a real valued function f a.e. Show that for every $\varepsilon > 0$ and $\delta > 0$, there exists a subset $A \subset E$ such that $m(A) < \delta$ and $n_0 \in nz$ such that for $n \geq n_0$ and $x \notin A$

$$|f_n(x) - f(x)| < \varepsilon \, .$$

(4) (25P) (Royden 3.30) Prove Egoroff's theorem: Let E be a measurable set of finite measure and (f_n) a sequence of measurable functions which converges to a real valued function f a.e. Show that on a set of large measure (f_n) converges uniformly to f.