

Real Analysis-Homework 9

Due date: Monday, November 15

- (1) (a) (10P) Let (x_k) be a sequence of real numbers such that

$$f((\alpha_k)) = \sum_{k=1}^n x_k \alpha_k$$

exists for all $(\alpha_k) \in \ell_2$. Show that $\sum_k |x_k|^2$ is finite.

- (b) (20P) Let X be a Banach space and $(f_n) \subset X^*$ be a sequence of linear functionals such that

$$f(x) = \lim_n f_n(x)$$

converges for all $x \in X$. Show that f is continuous.

- (2) (30P) Problem 18 in chapter 4 p=94 in Royden.
(3) (30P) Problem 19 in chapter 4 p=94 in Royden.