

Math 540-Real Analysis- Homework 1

**Due date:** September 9

- (1) Let  $R_2$  be the sets of finite disjoint union of rectangles of the form

$$(a, b] \times (c, d]$$

in the plane. Show that  $R_2$  is an algebra. Show that every triangle belongs to the  $\sigma$ -algebra generated by  $R_2$ . Show that

$$m((a, b] \times (c, d]) = (b - a)(d - c)$$

is  $\sigma$ -additive on  $R_2$ .

- (2) Problem 37-page (Show that the Cantor set obtained form iteratively removing a middle interval of length  $1/3$ , starting from  $[0, 1]$  is a compact set. Show also that every point  $x$  in the Cantor set can be written as

$$x = \sum_j a_j 3^{-j}$$

with  $a_j \in \{0, 2\}$ . What about uniqueness?)

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- (3) Problem 38-page 46. (Show that there is a bijection between the Cantor set and  $[0, 1]$ -and review bijection!)