Math 540-Real Analysis- Homework 2

Due date: September 17-**Submission in pairs**

(1) Let μ be an additive measure on an algebra $\mathcal{R}$. Assume in addition that μ is σ finite. Let Σ_μ be the algebra of measurable sets (constructed from the outer measure μ^*). Show that for every $B \in \Sigma_\mu$ there exists $A \in \mathcal{R}_{\sigma,\delta}$ such that $B \subset A$ and $\mu^*(A \setminus B) = 0$. Is the assumption μ σ-finite necessary?

(2) Show that for every set $B \subset \mathbb{R}$ with $m^*(B) > 0$ there exists a non-measurable set $E \subset B$.

(3) Let F be a monotone increasing function, $F(-\infty) = 0$ and F is right continuous, i.e. $\lim_{h\to 0, h>0} F(a+h) = F(a)$. Let $a_1 < a_2$ be such that $F(a_2) = F(a_1)$. Show that every set $A \subset (a_1, a_2)$ we have $\mu_F^*(A) = 0$. What about the endpoints?

(4) Let $\mathcal{R}$ be a σ-algebra and μ be an additive measure which is σ-additive on $\mathcal{R}$. Let μ_1 and μ_2 be two measures on $B(\mathcal{R})$ such that $\mu_1|\mathcal{R} = \mu_1|\mathcal{R}$. Show that $\mu_1 = \mu_1$. How can extension on Σ_μ differ?