Homework 3-Reak Analysis

Due date: September 24-sumbission in pairs

(1) Complete the proof of Lusin's theorem by showing the following facts

  i) Let $A \subset \mathbb{R}$ be a measurable set such that $m(A) < \infty$. Then there exists an open set $A \subset O$ such that $m(O \setminus A) < \varepsilon$.

  ii) Let (f_n) be sequence of continuous functions on a metric space (X, d) and $f : X \to \mathbb{R}$ satisfies

$$\lim_n \sup_{x \in X} |f_n(x) - f(x)| = 0$$

  Show that f is continuous.

(2) Problem 2.40 (Hint: First show that for every open set $O \subset (a, b)$ can be written in the form

$$O = \bigcup_j (a_j b_j)$$

with $(a_j b_j) \cap (a_k, b_k) = \emptyset$. In order to prove this you have to use that $\mathbb{R}$ admits a countable dense set.)