

1. Elementary Mathematics

Homework 1-Math 347-Junge

Due date: September, Friday 2.

Permanent homework: Find every new notion mentioned in class in the text book and read relevant pages about it ($\pm\epsilon$ pages)

(1) Negate the following statements

- i) Every cow is black.
- ii) For every black cow I can find a white cow.
- iii) $\forall x \in C \exists y \in D$,
- iv) $\forall x(x \in C \Rightarrow \exists y : y \in D)$.
- v) Sokrates car.
- vi) Sokrates is a liar.
- vii) On Crete every second person is a liar.
- viii) $\forall x \exists y \forall z \forall w \exists t \forall r(x < w) \Rightarrow (twz > yr)$.

(2) Let $A = \{1, 3, 5\}$, $B = \{2, 3, 5\}$ and $C = \{1, 2, 4\}$. Write down $A \cap B$, $(A \cap B) - C$, $(A \cap B) \cup C$, $(A \cup B) \cap C$, $(A \cap C) \cup (A \cap B)$, and the set $((A \cap B) - C) \cup ((B \cap C) - A) \cup ((C \cap A) - B)$. Give a formal proof of de Morgan's law (yes find the law in the book, shut the book and prove it).

(3) Let P, Q, R statements. Show that

- i) $(P \wedge Q) \Rightarrow (P \vee Q)$.
- ii) $((P \wedge Q) \vee P) \Rightarrow (\neg R \vee P) \Leftrightarrow \neg(\neg P \wedge R)$.

Hint: You may use truth tables or case by case analysis.

(4) Which of the following assignments are functions

i) $f : \{1, 2, 3\} \rightarrow \{1, 2, 3\}$ given by

$$(1, 2), (2, 4), (2, 4) .$$

ii) $f : \{1, 2, 3\} \rightarrow \{1, 2, 3\}$ given by

$$(1, 2), (2, 4), (3, 4) .$$

iii) $f : \{1, 2, 3\} \rightarrow \{1, 2, 3\}$ given by

$$(1, 2), (2, 4), (3, 2^2 - 4) .$$

iv) $f : \{1, 2, 3\} \rightarrow \{1, 2, 3\}$ given by

$$(1, 2), (2, 4), (3, 2^2 - 1) .$$

v) $f : \mathbb{N} \rightarrow \mathbb{N}$ where $f(x)$ is the third number after x .

- vi) $f : \mathbb{N} \rightarrow \mathbb{N}$ where $f(x)$ is the $f(x - 1)$ -th number after x .
 - vii) $f : \mathbb{R} \rightarrow \mathbb{R}$ where $f(x)$ is the third number after x .
 - viii) $f : \mathbb{R}^2 \rightarrow \mathbb{R}$, $f(x, y)$ is the smallest solution to $(3x + 4y)^2 + z^2 = 6$.
 - ix) $f : \mathbb{R}^2 \rightarrow \mathbb{R}$, $f(x, y)$ is the smallest solution to $(3x + 4y)^2 + z^2 = 6 + 21x^2 + 28y^2$. (Hint: You may use $|xy| \leq \frac{x^2}{2} + \frac{y^2}{2}$ for real numbers).
- (5) 1.17 on page 21.
- (6) 1.20 on page 22.
- (7) 2.1. on page 44.