Math 406-History of Calculus- Homework 4

Due date September 23.

1) Consider Archimedes approximation for π with a regular polygon with n sides. Show that

$$t_n = \tan(\pi/n) \ ,$$
$$s_n = 2\sin(\pi/n) \ ,$$
$$o_n + m_n = \frac{\tan(\pi/n)+\sin(\pi/n)}{2}(1-\cos(\pi/n)) \ .$$

What means that nt_n approaches π. Use this to determine

$$\lim_n n\sin(\pi/n) \ .$$

2) Complete the proof of $a(D(r)) = \frac{r}{2}|\partial D(r)|$. The missing case is $a(D(r)) > \frac{1}{2}r|\partial D(r)|$.

3) Let C be cone with basis r and slanted height s. We want to show that

$$|\partial C_{up}| \ \geq \ s|\partial D(r)| \ .$$

Here $|\partial C_{up}|$ is the surface area of the upper part so that

$$|\partial C| \ = \ a(D(r)) + |\partial C_{up}| \ .$$

Proceed in the following steps

i) Assume that $|\partial C_{up}| < s|\partial D(r)|$. Find a regular polygon $P_n(r) \subset D(r)$ such that

$$|\partial C_{up}| < s|\partial P_n(r)| \ .$$

ii) Observe that

$$|\partial C| < a(D(r)) + s|\partial P_n(r)|$$

find a regular polygon $P_m(r) \subset D(r)$ such that

$$|\partial C| < a(P_m(r)) + s|\partial P_n(r)|$$

iii) Join all the edges and conclude.

In your argument don't forget to mention what you use.