

K-theory -Hw 1

**Due date:** Monday, November 2.

i) Calculate

$$\sqrt{\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}}$$

and

$$\sqrt{\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}}$$

ii) Let  $S : \ell_2(\mathbb{N}) \rightarrow \ell_2(\mathbb{N})$  be the linear map given by

$$S(e_j) = e_{j-1}$$

for  $j > 1$  and  $S(e_1) = 0$ . Show that for no  $n \times n$  matrix  $A$  the map  $A + S$  is invertible.

iii) Find an example of a surjective  $*$ -homomorphism  $\pi : A \rightarrow B$  such that  $u \in B$  is a unitary, but there is no unitary  $w \in A$  with  $\pi(w) = u$ . Why is this not possible for  $A$  and  $B$  finite dimensional.