Mon, 10/12/09

ORESME $\sum_{j=1}^{\infty} \frac{j}{2^j} = 2$ $\leftarrow$ prove this using

$$\sum_K \sum_j a_{jK} = \sum_j \sum_K a_{jK} \qquad a_{jK} \geq 0$$

Hint: $j = \sum_{K=1}^{j} 1$

Proof

$$\sum_{j=1}^{\infty} j 2^{-j} = \sum_{j=1}^{\infty} \sum_{K=1}^{j} 2^{-j}$$

$$a_{jK} = \begin{cases} 2^{-j} & K \leq j \\ 0 & K > j \end{cases}$$

$$= \sum_{j=1}^{\infty} \sum_{K=1}^{\infty} a_{jK}$$

$$= \sum_{K=1}^{\infty} \sum_{j=1}^{\infty} a_{jK}$$

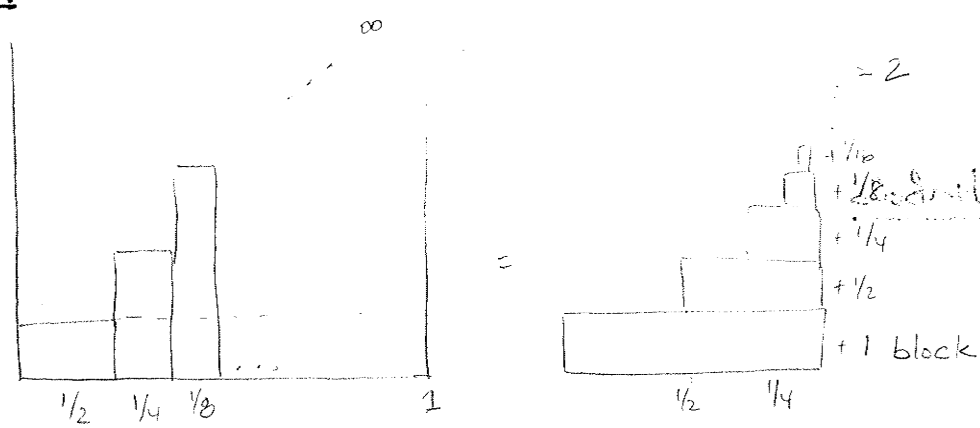
fix K

$$\sum_{j=1}^{\infty} a_{jK} = \sum_{j=K}^{\infty} 2^{-j} = 2^{-K}$$

$$= \sum_{K=1}^{\infty} \underbrace{\left(\sum_{j=K}^{\infty} 2^{-j} \right)}$$

$$= \sum_{K=1}^{\infty} 2^{-K} = 2$$

Oresme's Proof



This is different from [illegible] by using infinity.

Theorem $\frac{a}{K} \sum_{j=0}^{\infty} (n - \frac{1}{K})^j = \frac{a}{K} \frac{1}{1-(1-\frac{1}{K})} = a$

Oresme also introduced
- functional relations
- graphic illustrations of thos

Merton's Rule

- In Merton's school in Oxford velocity and motion was discussed.

Aristotele "challenged" scientists to explain

- infinity
- continuity
- instantaneous velocity

Merton's Rule

- constant acceleration a

v_0 initial velocity

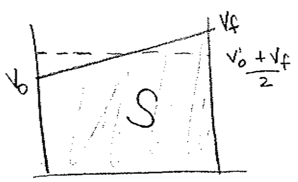
v_f final velocity

s distance traveled

t time

$$\boxed{S = \frac{1}{2}(v_0 + v_f)t}$$

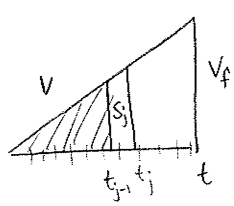
Oresme's Picture



"distance and integration - area"

The Law of odd Numbers

$V_0 = 0$



$t_j = \frac{j}{n}T$

S_j = distance traveled at time t_j

$\Delta_j = S_j - S_{j-1}$

= distance traveled between velocity t_{j-1} and t_j

$$\Delta_j = \frac{1}{2}at_j^2 - \frac{1}{2}at_{j-1}^2$$

$$= \frac{1}{2}a\left(\frac{j^2}{n^2}T^2 - \frac{(j-1)^2}{n^2}T^2\right)$$

$$= \frac{1}{2}a\frac{T^2}{n^2}\left(j^2 - (\cancel{j^2} - 2j + 1)\right)$$

$$= \frac{1}{2}a\frac{T^2}{n^2}(2j-1)$$

Decartes introduced x, y variables
powers x^a, y^k

$\not\exists$ this =

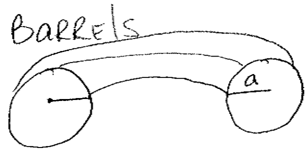
missing $+, -$ (German)
P M

Infinitesimals and Early Indivisible

Kepler

- Wine Barrels

Ex.



donut

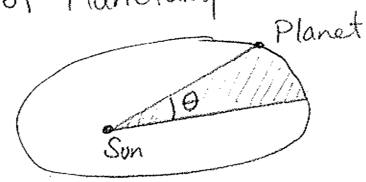
$$\text{Volume} = \pi a^2 (2\pi b)$$

"Proof": take a slice.

$$\text{area(slice)} = \pi a^2 t \qquad t = \frac{t_1 + t_2}{2} \qquad \begin{aligned} t_1 &= b - a \\ t_2 &= b + a \end{aligned}$$

$$\text{Volume} = \left(\Sigma\, t\right) \pi a^2$$
$$= (2\pi b)\, \pi a^2$$

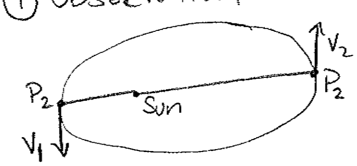
Laws of Planetary Motion



$$\frac{d\,A(\theta)}{d\,\theta} = 0$$

Johannes Proof

① observation



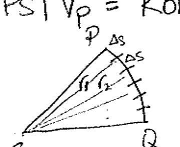
P_1 closest to Sun

P_2 farthest away sun

$$|P_1 S|\, V_1 = |P_2 S|\, V_2$$

② $|PS|\, V_P$ = Konstant for all positive (false)

③



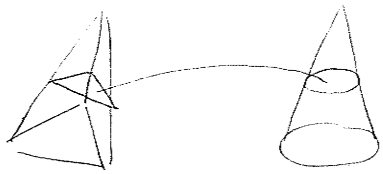
Δs distance traveled

$$t = \Sigma\, t_i \simeq \Sigma \frac{\Delta s}{V_i} = \tilde{K} \left(\sum_i r_i \, \Delta s \right)$$ ← False for ellipse

$\parallel$ Area $(\overset{\frown}{SPQ})$

However, the result is true.

Cavaleri



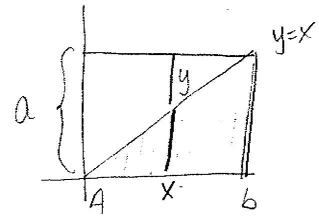
AREAS same at each level
=> Volume is same.

E AREA(△) = area (○)

Fri, 10/16/09

Cavaleri

$$\int_0^1 x^k\,dx = \frac{1}{k+1}$$



$B-A=a$

$\sum_A^B x = ?$

symmetry

$$a^2 = \sum_A^B a = \sum_A^B (x+y) = \sum_A^B x + \sum_A^B y \overset{\swarrow}{=} 2\sum_A^B x$$

Result $\boxed{\sum_A^B x = \frac{a^2}{2}}$ $\left(\int_0^a x\,dx = \frac{a^2}{2}\right)$

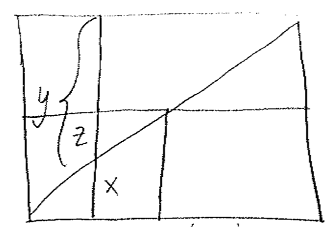
Cavaleri carried on to show

$\sum_A^B x^k = \frac{a^{k+1}}{k+1}$ for $k = 1, \ldots, 9$

$\sum_A^B x^2 = ?$ $\quad a^3 = \sum_A^B a^2 = \sum_A^B (x+y)^2 = \sum_A^B x^2 + 2xy + y^2$

$= \sum_A^B x^2 + \sum_A^B y^2 + \sum_A^B 2xy = 2\sum_A^B x^2 + 2\sum_A^B xy$

$= 2\sum_A^B x^2 + 4\sum_A^{B/2} xy$



$\frac{B}{2}\left(=\frac{a}{2}\right)$

Define $z = \frac{a}{2} - x$

observe $\left(\frac{a}{2}+z\right)\left(\frac{a}{2}-z\right)$

$= y \cdot x$

Hence $xy = \frac{a^2}{4} - z^2$

$$= 2\sum_A^B x^2 + \underbrace{4\sum_A^{B/2} \frac{a^2}{4}}_{\frac{a^3}{2}} - 4\sum_A^{B/2} z^2$$

By symmetry, to z^2 — also 3-D

Hence

$$\frac{a^3}{2} = 2\sum_A^B x^2 - 4\sum_A^{B/2} x^2 = 2\sum_A^B x^2 - \frac{4}{8}\sum_A^B x^2 = \frac{3}{2}\sum_A^B x^2$$

$\boxed{\text{Hence, } \frac{a^3}{3} = \sum_A^B x^2}$

RIGOROUS VERSION

① $\sum x = \frac{a^2}{2}$

$$n = \sum_{K=1}^{n} 1 = \sum_{K=1}^{n} \left(\overset{x}{\frac{K}{n}} + \overset{y}{\frac{n-K}{n}} \right) = \sum_{K=1}^{n} \frac{K}{n} + \sum_{K=1}^{n} \frac{n-K}{n}$$

$$= \sum_{K=1}^{n} \frac{K}{n} + \sum_{K=1}^{n-1} \frac{K}{n}$$

$$2 \sum_{K=1}^{n-1} K < n^2 \leq 2 \sum_{K=1}^{n} K$$

$\Rightarrow$ AREA $(\Delta) = \frac{1}{2}$

Notation: $a_n = b_n + \theta(n^a)$. If $|a_n - b_n| \leq C n^a$

$$a^3 = \sum_A^B a^2 = \sum_A^B (x+y)^2 = \sum_A^B (x^2 + y^2 + 2xy)$$

$$n^3 = \sum_{K=1}^{n} (K + (n-K))^2$$

$$= \sum K^2 + \sum (n-K)^2 + 2 \sum K(n-K)$$

$$= 2 \sum_{K=1}^{n} K^2 + 4 \sum_{K=1}^{n/2} K(n-K) + \theta(n^2)$$

$$= 2 \sum_{1}^{n} K^2 + 4 \sum_{1}^{n/2} \frac{n^2}{4} - Z_K^2$$

$$= 2 \sum_{1}^{n} K^2 + \frac{n^3}{2} - 4 \underbrace{\sum_{K=1}^{n} \left(\frac{n}{2} - K\right)^2}_{\text{Lemma}} + \theta(n^2)$$

$$= 2 \sum_{1}^{n} K^2 + \frac{n^3}{2} - \frac{4}{8} \sum_{1}^{n} K^2 + \theta(n^2)$$

$Z_K = \frac{n}{2} - K$

$K = \frac{n}{2} - Z_K$

$n - K = \frac{n}{2} + Z_K$