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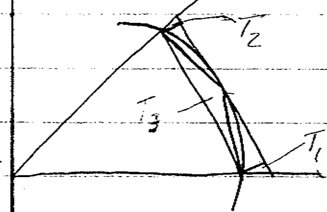
MTH 4106
9/9/09
HW 2

perfect

(1) 

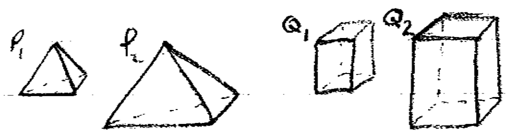
To show that for every disc D, we can find regular polygons Q_n containing the discs such that $a(Q_n) - a(D)$ becomes arbitrarily small, it is sufficient to show that $a(\;) \le 4a(\;) \le 4a(\;)$. It is obvious that $4a(\;) \le 4a(\;)$, since the triangle is always contained within the area between the edge of the inscribed polygon and the circle's edge. T_2 is half the area of the rectangle in the picture ( ), and T_3 is clearly bigger than [illegible], so $4a(T_3) > a(T_1) + a(T_2) + 2(T_3) > a(\;)$. Since we know from class that $a(\;)$ becomes arbitrarily small, $4a(\;)$ must also become arbitrarily small. Since $a(\;) \le 4a(\;)$, $a(\;)$ must also become arbitrarily small. ∎

20 pt

(2) p.22 #19 A convex polygon can be broken into m triangles by choosing an arbitrary point contained in the polygon and connecting all vertices of the polygon to the point, assuming the polygon has m edges. Let $\frac{1}{n_x}$ be the fraction of the area of the base, A, taken up by the xth triangle. Then the volume of the triangular pyramid made by the xth triangle will be $\frac{1}{3}\frac{A}{n_x}h$, since the [illegible] of the original pyramid will be shared by all the new triangular pyramids, so the height of all the pyramids will be the same. Since we know that the entire base can be dissected into triangles, the volume of the original pyramid can be found by adding the volumes of the triangular pyramids: $\frac{1}{3}\frac{A}{n_1}h + \frac{1}{3}\frac{A}{n_2}h + \dots + \frac{1}{3}\frac{A}{n_m}h = \frac{1}{3}h\sum_{x=1}^{m}\frac{A}{n_x}$. We know

that $\sum_{x=1}^{n} \frac{A}{n_x} = A$, since the triangles composing the base cover the base exactly once, so the summation of their areas is A. Therefore, $\frac{1}{3}h \sum_{x=1}^{n} \frac{A}{n_x} = \frac{1}{3} h A =$ $= \frac{1}{3} Ah$, so the volume formula holds. ∎ 10P

(3)
p. 23 #20



Let P_1, P_2 be similar pyramids and Q_1, Q_2 be similar rectangular prisms. Let h_1 be the height of P_1 and Q_1, and let h_2 be the height of P_2 and Q_2. Let A_1 be the area of the base of P_1 and Q_1, and let A_2 be the area of the bases of P_2 and Q_2. We know that we can fit three of each pyramid into it's corresponding rectangular prism ($P_1 \to Q_1$, $P_2 \to Q_2$), so $3vol(P_1) = vol(Q_1)$ and $3vol(P_2) = vol(Q_2)$. Since Q_1 and Q_2 are rectangular prisms, we know that $(vol(Q_1) : vol(Q_2)) = (h_1^3 : h_2^3)$, since h_1 and h_2 are each the length of one of the sides of their respective prisms. Substituting, we see that $(3vol(P_1) : 3vol(P_2)) = (h_1^3 : h_2^3)$. Since we are dealing with ratios, we can drop the 3's from the left side of the equation, since there is a 3 on each portion of the ratio. This gives us $(vol(P_1) : vol(P_2)) = (h_1^3 : h_2^3)$. Since all one-dimensional measurements are different by the same factor in similar figures, we can substitute in any one-dimensional length for h_1 and h_2. We'll choose s_1 and s_2, any arbitrary corresponding edges in P_1 and P_2 respectively. This gives us $(vol(P_1) : vol(P_2)) = (s_1^3 : s_2^3)$ as desired. ∎

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