

Transition course -hw1

Due date: Wednesday, September 8

- (1) Show that a bounded, monotone sequence is convergent.
- (2) Give an ε - δ proof for

$$\lim_n \left| \frac{5n + n^2}{n^2 - 5n} \right| = 1.$$

- (3) Let $x > 0$ show that

$$\lim_n x^{\frac{1}{n}} = 1.$$

(Hint: you may use the continuous function $f(x) = \ln x$ or Bernoulli's inequality.)

- (4) Show that the vectors $x_1, \dots, x_n \in \mathbb{R}^n$ given by

$$x_j = (1, 1, \dots, \underbrace{1}_{j\text{-th position}}, 0, 0, \dots, 0)$$

are linearly independent.

- (5) Show that $A = \{x \in \mathbb{R} : \exists_{k \in \mathbb{N}} 2k \leq x \leq 2k + 1\}$ is closed.
- (6) Use the ε - δ criterion to show that $f : [0, \infty) \rightarrow [0, \infty)$ given by $f(x) = \sqrt{x}$ is continuous.