Transition course -hw5

Due date: Monday, October 4

(1) On $X = \{-1,1\}^{\mathbb{N}} = \{(\varepsilon_1, \varepsilon_2, \cdots) \ : \ \varepsilon_i = \pm 1\}$ we define the metric

$$d((\varepsilon_i),(\delta_i)) \ = \ \sum_{i=1}^{\infty} 2^{-i}|\varepsilon_i - \delta_i| \ .$$

Show that (X,d) is a compact metric spaces. Show that

$$f((\varepsilon_i)) \ = \ \sum_i \alpha_i \varepsilon_i$$

is continuous if and only if $\sum_i |\alpha_i| < \infty$.

(2) We consider the space $X = C[0,1]$ and

$$F \ = \ \{t \mapsto \sum_{k=0}^{n-1} a_k t^k \ : \ \sum_{k=0}^{n-1} |a_k| \ \leq \ 1\}$$

Show that F is compact by finding a compact set $C \subset \mathbb{R}^n$ and a continuous function $g : C \to C[0,1]$ such that $F = f(C)$.