

Ryan

1. Problem 1: (8 points)

Find

$$\lim_{x \rightarrow 0} \frac{x^4 - x^2}{1 - \cos x}$$

giving justifications of each of your steps.

$$\lim_{x \rightarrow 0} \frac{x^4 - x^2}{1 - \cos x} \overset{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{4x^3 - 2x}{\sin x} \overset{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{12x^2 - 2}{\cos x} = \frac{-2}{1} = -2$$

(1)  $\frac{0}{0}$  form  
 (2)  $\frac{0}{0}$  form  
 (3) standard limit rules, or continuity

(1) L'H ≠ QR

-3  $\sin 0 = 1$  Note: listing L'H rather than  $\frac{0}{0}$  as justifi'n is fine too!

-3  $\frac{d}{dx}(1 - \cos x) = 0 + \sin x$   $\frac{0}{0} = 0$

-1/2 no lim  
 -2 what's no lim

-3  $\frac{3}{0} = \frac{10}{0} = \frac{24x}{-24} = \frac{24}{-24} = -24$

-4  $\frac{-5}{-3} \frac{0}{0} = 0$

L'H ≠ QR

2. Problem 2: (10 points)

- (a) (3 points) Give a precise statement of the Extreme Value Theorem.
- (b) (7 points) Find the absolute maximum of the function

$$f(x) = x^3 + 3x^2$$

on the interval  $[-4, 4]$ .

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$x$	-4	-2	0	4
$f(x)$	-16	4	0	112

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- (a) (3 points) Give a precise statement of the Extreme Value Theorem.  
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on the interval  $[-4, 4]$ .

a) EVT: A continuous function on a closed interval has an absolute max and an absolute min (on that interval).

b)  $f$  is a polynomial, so critical numbers are where  
 $f'(x) = 3x^2 + 6x = 3x(x+2) = 0, \Rightarrow x = 0, -2$

Then  $f(-4) = -64 + 48 = -16$

$f(-2) = -8 + 12 = 4$

$f(0) = 0$

$f(4) = 64 + 48 = 112 \leftarrow \text{abs. max}$