

1. Problem 1: (10 points)

Find the following antiderivatives:

(a) (1 points)

$$\int \sin x dx = -\cos x + C$$

(b) (2 points)

$$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

(c) (3 points)

$$\int \frac{dx}{x^{17}} = \int x^{-17} dx = \frac{x^{-16}}{-16} + C = -\frac{1}{16x^{16}} + C$$

(d) (4 points)

$$\int (3 \sec^2 x + \frac{4}{x}) dx = 3 \int \sec^2 x dx + 4 \int \frac{dx}{x} = 3 \tan x + 4 \ln|x| + C$$

2. Problem 2: (10 points)

(a) (6 points) Approximate the area between  $y = x^2$  and the  $x$ -axis for  $0 \leq x \leq 2$  using  $n$  rectangles and a right endpoint calculation. Express your answer as a function of  $n$ , evaluating any summations that arise.

(b) (4 points) What is the exact area of the region described in part a)? Compute its value in two different ways.

a)  $\Delta x = \frac{2-0}{n} = \frac{2}{n}$  <sup>1 pt</sup>,  $x_0 = 0$ , so  $x_i = \frac{2i}{n}$ . <sup>1 pt</sup>  
 Then approx area =  $\sum_{i=1}^n (\frac{2i}{n})^2 \cdot \frac{2}{n} = \frac{8}{n^3} \sum_{i=1}^n i^2 = \frac{8}{n^3} \frac{n(n+1)(2n+1)}{6}$  <sup>2 pt</sup>  
 $= \frac{4}{3} \frac{(n+1)(2n+1)}{n^2}$  <sup>2 pt</sup>

b) Method 1: Area =  $\lim_{n \rightarrow \infty} \frac{4}{3} \frac{(n+1)(2n+1)}{n^2} = \lim_{n \rightarrow \infty} \frac{4}{3} (1 + \frac{1}{n})(2 + \frac{1}{n}) = \frac{8}{3}$  <sup>2 pt</sup>

Method 2: Area =  $\int_0^2 x^2 dx = \frac{x^3}{3} \Big|_0^2 = \frac{8}{3} - 0 = \frac{8}{3}$  <sup>2 pt</sup>