Math 220 Quiz # 1 Solutions

1. Find the domain of the function $f(x) = \sqrt{2x^2 - 32}$ and express it as a union of intervals.
   Solution: The domain consists of all x for which

$$\begin{aligned} 2x^2 - 32 &\geq 0 \\ \text{equivalently } x^2 &\geq 16 \\ \text{equivalently } |x| &\geq 4 \\ \text{equivalently } x &\in (-\infty, -4] \cup [4, \infty) \end{aligned}$$

2. For the function $g(x) = e^{\sqrt{x+1}}$, find an expression for the inverse function $g^{-1}(x)$. State the range of g^{-1}.
   Solution: The domain of g is all x for which $x + 1 \geq 0$, i.e. $x \geq -1$, so the range of g^{-1} is $[-1, \infty)$, i.e. same as the domain of g. The inverse is computed as follows:

$$\begin{aligned} y &= e^{\sqrt{x+1}} \\ \text{(interchange } x \text{ and } y\text{)} \quad x &= e^{\sqrt{y+1}} \\ \text{(equivalently)} \quad \ln x &= \sqrt{y+1} \\ \text{(equivalently)} \quad (\ln x)^2 &= y + 1 \\ \text{(equivalently)} \quad (\ln x)^2 - 1 &= y \\ \text{(thus)} \quad g^{-1}(x) &= (\ln x)^2 - 1 \end{aligned}$$

3. Simplify the function $f(x) = 2\sin(\tan^{-1} x)$ by writing it in an alternate but equivalent form without the use of trig or inverse trig functions.
   Solution: Set $y = \tan^{-1} x$, so that $\tan y = x$ and $f(x) = 2\sin y$. Now construct a right-triangle with angle y, adjacent side 1 and opposite side x. For this triangle we have $\tan y = \text{opposite}/\text{adjacent} = x/1 = x$ as desired. The hypotenuse of the triangle is $\sqrt{x^2 + 1}$. Therefore we read from the triangle that $\sin y = \text{opposite}/\text{hypotenuse} = x/\sqrt{x^2 + 1}$. But $f(x)$ is just twice this:

$$f(x) = \frac{2x}{\sqrt{x^2 + 1}}$$