Math 220 Quiz # 7 - Solutions

1. Approximate the area under the curve $y = x^2 + 8$ on the interval $[0, 1]$ using 16 rectangles and using the left endpoint of each subinterval as the evaluation point.
   Solution:
   The approximation we need is the Riemann sum

$$A_{16} = \sum_{i=1}^{16} (x_{i-1}^2 + 8)\Delta x$$

   where x_{i-1} is used rather than x_i since we want left endpoints. The interval is $[0, 1]$, so

$$\begin{aligned} \Delta x &= \frac{1}{16} \\ x_i &= 0 + i\Delta x = \frac{i}{16} \end{aligned}$$

   Thus

$$\begin{aligned} A_{16} &= \sum_{i=1}^{16} \left(\left(\frac{i}{16} \right)^2 + 8 \right) \frac{1}{16} \\ &= \frac{1}{16} \sum_{i=1}^{16} \left(\left(\frac{1}{16} \right)^2 i^2 + 8 \right) \\ &= \frac{1}{16^3} \sum_{i=1}^{16} i^2 + \frac{8}{16} \sum_{i=1}^{16} 1 \\ &= \frac{1}{16^3} \frac{16(17)(33)}{6} + \frac{8}{16} 16 = 8 + \frac{17 \times 11}{2 \times 16^2} = \frac{4283}{512} \end{aligned}$$

2. Compute

$$\int_1^{16} (\sqrt{x} - \frac{1}{x^2}) dx$$

   Solution:
   By the power rule and the fundamental theorem

$$\begin{aligned} \int_1^{16} (\sqrt{x} - \frac{1}{x^2}) dx &= \int_1^{16} (x^{1/2} - x^{-2}) dx \\ &= \left(\frac{x^{3/2}}{3/2} - \frac{x^{-2+1}}{-2+1} \right) \Bigg|_1^{16} \\ &= \left(\frac{2}{3} 16^{3/2} + 16^{-1} \right) - \left(\frac{2}{3} 1^{3/2} + 1^{-1} \right) \\ &= \left(\frac{2}{3} 64 + \frac{1}{16} \right) - \left(\frac{2}{3} + 1 \right) = \frac{657}{16} \end{aligned}$$

3. Find the derivative $f'(x)$ of the function

$$f(x) = \int_1^{\sin x} e^{-t^2} dt$$

Solution:
By the Chain Rule and the Fundamental Theorem

$$f'(x) = e^{-(\sin x)^2} (\sin x)' = (\cos x) e^{-\sin^2 x}$$