

Assignment 1

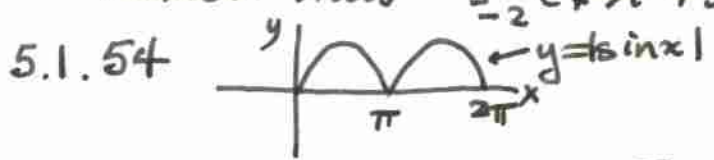
- § 5.1: 46, 48, 52, 54, 64, 66, 70
- § 5.2: 52, 54, 58
- § 5.3: 38, 44, 50, 52, 56, 58
- § 5.4: 24, 26, 28, 32, 34, 52, 70, 74
- § 5.6: 16
- § 5.7: 16, 30, 38

Solutions/Hints:

5.1.46 $\int_0^4 (3-2f(x)) dx = \int_0^4 3 dx - 2 \int_0^4 f(x) dx = 3x \Big|_0^4 - 2 \left(\int_0^2 f dx + \int_2^4 f dx \right)$
 $= 12 - 0 - 2(2+1) = 12 - 18 = -6$

5.1.48 If $f(x) < 3$ in $[2, 4]$, then $\int_2^4 f(x) dx \leq 3(4-2) = \text{area of rectangle above } f \text{ on } [4, 2], \text{ i.e.}$
 $\int_2^4 f(x) dx \leq 6$. This contradicts the given $\int_2^4 f(x) dx = 7$.

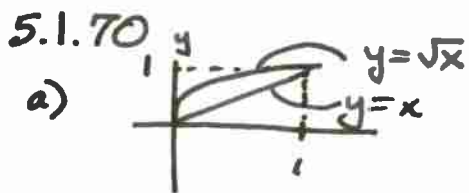
5.1.52 $f(x) = 7x^5 + 6$ is an odd fn and so the graph for negative x is the negative of the graph for positive x . This means that the integrals from -2 to 0 and 0 to 2 cancel. Thus $\int_{-2}^2 (7x^5 + 3) dx = 0$.



So $\int_0^{2\pi} |\sin x| dx = 2 \int_0^{\pi} \sin x dx = 4$

5.1.64 Using a graphing calculator we find that $2.29 \leq e^x \sin x \leq 7.45$ for $1 \leq x \leq 3$
 Thus $2.29 \times 2 \leq \int_1^3 e^x \sin x dx \leq 7.45 \times 2$
 and so, in particular, $4.5 \leq \int_1^3 e^x \sin x dx \leq 15$

5.1.66 $\frac{1}{1} \int_0^1 f(x) dx = 2, \frac{1}{2} \int_0^3 f(x) dx = 4$. These are given.
 Thus $\frac{1}{3} \int_0^3 f(x) dx = \frac{1}{3} \left[\int_0^1 + \int_1^3 \right] = \frac{1}{3} [2 + 8] = \frac{10}{3}$



b) Area = $\int_0^1 \sqrt{x} dx - \int_0^1 x dx$

5.2.52 Since $H'(x) = f(x)$, H is increasing where f is non-negative i.e. $-5 \leq x \leq -3$ and $1 \leq x \leq 5$

5.2.54 To the left of -3 we are adding +ve area to H , so H is increasing. To the right of -3 we are adding -ve area to H , so H is decreasing. Hence there is a local max at $x = -3$.

5.2.58 $\int_{\sqrt{\pi/2}}^x = \int_0^x + \int_{\sqrt{\pi/2}}^0 = \int_0^x - \int_0^{\sqrt{\pi/2}} = \sin(x^2) - \frac{1}{\sin(\pi/2)}$

5.3.38 Since $F'(x) = f(x)$ and $F''(x) = f'(x)$, F is concave up where $f'(x) > 0$, i.e. on $(2, 4)$ and $(6, 8)$

5.3.44 By FTC, $g(w) = \frac{d}{dw}(\cos(\pi e^w)) = -\sin(\pi e^w) \cdot \pi e^w$

so $g(0) = -\sin(\pi e^0) \cdot \pi e^0 = -\pi \sin \pi = 0$

5.3.50 Let $H(x) = \int_2^x e^{-t^2} dt$. Then $G(x) = H(x^3)$. By the chain rule $G'(x) = H'(x^3) \cdot 3x^2 = 3x^2 e^{-x^6}$

5.3.52 Average value = 1 means that

$$\frac{1}{5+2} \int_{-2}^5 g'(x) dx = 1$$

But this integral is $g(5) - g(-2)$. Thus

$$g(5) - g(-2) = 7 \Rightarrow g(5) = 7 + g(-2) = 7 + 3 = 10$$

5.3.56 By FTC, $g'(x) = f(x)$, so $g'(2) = f(2) = 0$. Thus there is a horizontal tangent line at $x = 2$.

5.3.58 average value = $\frac{1}{3} \int_0^3 g'(x) dx = \frac{1}{3} (g(3) - g(0)) = \frac{1}{3} \int_1^3 f' - \frac{1}{3} \int_1^0 f' = \frac{1}{3} \int_0^3 f(x) dx$
Estimations from the graph show that this value is negative!

5.4.24 $u = 2 - 3x$, $du = -3 dx$, $\int \sin(2 - 3x) dx = -\frac{1}{3} \int \sin u du = \frac{1}{3} \cos u + C = \frac{1}{3} \cos(2 - 3x) + C$

5.4.26 $u = 1 - x^2$, $du = -2x dx$, $\int x \cos(1 - x^2) dx = -\frac{1}{2} \int \cos u du = -\frac{1}{2} \sin u + C = -\frac{1}{2} \sin(1 - x^2) + C$

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5.4.28 $u = 3x+2 \Rightarrow x = \frac{u-2}{3}, dx = \frac{1}{3} du$

$$\int x(3x+2)^4 dx = \frac{1}{3} \int \frac{u-2}{3} u^4 du = \frac{1}{9} \int (u^5 - 2u^4) du = \frac{1}{9} \left(\frac{u^6}{6} - \frac{2u^5}{5} \right) + C$$

$$= \frac{1}{9} \left(\frac{(3x+2)^6}{6} - \frac{2(3x+2)^5}{5} \right) + C$$

5.4.32 $u = \ln x \Rightarrow du = \frac{dx}{x}$ so $\int \frac{\ln x}{x} dx = \int u du = \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$

5.4.34 $u = x^2 \Rightarrow du = 2x dx$ so $\int x e^{x^2} dx = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$

5.4.52 $I = \int \sec x \tan x dx = \int \frac{1}{\cos x} \frac{\sin x}{\cos x} dx = \int \frac{\sin x}{\cos^2 x} dx$

Set $u = \cos x \Rightarrow du = -\sin x dx$, so

$$I = -\int \frac{du}{u^2} = \frac{1}{u} + C = \frac{1}{\cos x} + C = \sec x + C$$

5.4.70 $u = 8-x \Rightarrow du = -dx, u = 8 - (-19) = 27$ when $x = -19$,

$u = 8 - 8 = 0$ when $x = 8$. Thus

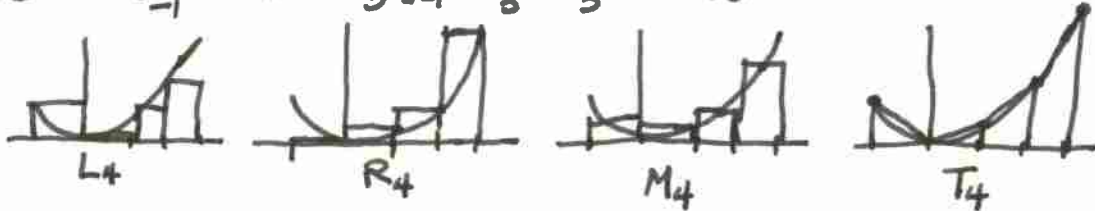
$$\int_{-19}^8 \sqrt{8-x} dx = -\int_{27}^0 \sqrt{u} du = \int_0^{27} u^{1/2} du = \left[\frac{2}{3} u^{3/2} \right]_0^{27} = \frac{2}{3} \cdot 27\sqrt{3} = 54\sqrt{3}$$

5.4.74 $u = \cos x \Rightarrow du = -\sin x dx, u = \cos(-\frac{\pi}{2}) = 0$ when $x = -\frac{\pi}{2}$,

$u = \cos \pi = -1$ when $x = \pi$, so

$$\int_{-\pi/2}^{\pi} e^{\cos x} \sin x dx = -\int_0^{-1} e^u du = \int_{-1}^0 e^u du = \left[e^u \right]_{-1}^0 = 1 - \frac{1}{e}$$

5.6.16 $I = \int_{-1}^3 x^2 dx = \left[\frac{x^3}{3} \right]_{-1}^3 = \frac{27}{3} - \frac{-1}{3} = 9\frac{1}{3}$



$L_4 = 1^2 \cdot 1 + 0^2 \cdot 1 + 1^2 \cdot 1 + 2^2 \cdot 1 = 6$, $R_4 = 0^2 \cdot 1 + 1^2 \cdot 1 + 2^2 \cdot 1 + 3^2 \cdot 1 = 14$

$M_4 = \left(-\frac{1}{2}\right)^2 \cdot 1 + \left(\frac{1}{2}\right)^2 \cdot 1 + \left(\frac{3}{2}\right)^2 \cdot 1 + \left(\frac{5}{2}\right)^2 \cdot 1 = \frac{36}{4} = 9$, $T_4 = \frac{L_4 + R_4}{2} = \frac{6 + 14}{2} = 10$

5.7.16 a) Here $x_i = a + i \frac{b-a}{n} = 1 + i \frac{1-1}{10} = 1+i$, $\Delta x = \frac{b-a}{n} = 1$

Thus $R_{10} = \sum_{i=1}^{10} x_i^2 \Delta x = \sum_{i=1}^{10} (1+i)^2$

b) $R_5 = \sum_{i=1}^5 (1+2i)^2 \cdot 2$, c) $R_{20} = \sum_{i=1}^{20} \left(1 + \frac{i}{2}\right)^2 \cdot \frac{1}{2}$, d) $R_{50} = \sum_{i=1}^{50} \left(1 + \frac{i}{5}\right)^2 \cdot \frac{1}{5}$

5.7.30 With $f(x) = \sqrt{x}$, $a=1, b=4$ we have $x_k = a + k \frac{b-a}{n} = 1 + \frac{3k}{n}$. Thus the sum in the problem is R_n and this approaches the integral as $n \rightarrow \infty$ since it is a Riemann sum.

5.7.38

$$\int_0^1 x^2 dx$$

$x_i = 0 + i \frac{1-0}{n} = \frac{i}{n}$. Then $m_i = \frac{x_{i-1} + x_i}{2} = \frac{\frac{i-1}{n} + \frac{i}{n}}{2} = \frac{2i-1}{2n}$. $\Delta x = \frac{1}{n}$.

$$\begin{aligned}
 \therefore M_n &= \sum_{i=1}^n m_i^2 \Delta x = \frac{1}{n} \sum_{i=1}^n \left(\frac{2i-1}{2n} \right)^2 = \frac{1}{4n^3} \sum_{i=1}^n (4i^2 - 4i + 1) \\
 &= \frac{1}{4n^3} \left(4 \sum_{i=1}^n i^2 - 4 \sum_{i=1}^n i + \sum_{i=1}^n 1 \right) \\
 &= \frac{1}{4n^3} \left(\frac{4n(n+1)(2n+1)}{6} - 4 \frac{n(n+1)}{2} + n \right) \\
 &= \frac{1}{6} \left(1 + \frac{1}{n} \right) \left(2 + \frac{1}{n} \right) - \frac{1}{2n} \left(1 + \frac{1}{n} \right) + \frac{1}{4n^2} \rightarrow \frac{1}{6} \cdot 1 \cdot 2 - 0 + 0 = \frac{1}{3} \\
 &\quad \text{as } n \rightarrow \infty.
 \end{aligned}$$