Assignment 2

Sec 6.1: 4, 20, 61, 62, 63, 64

Sec 6.2: 8, 10, 12, 16

6.1.4 $f(x) = \cos(\frac{1}{x}) \Rightarrow f'(x) = \frac{-\sin(\frac{1}{x})}{-x^2} = \frac{1}{x^2}\sin(\frac{1}{x})$

a) for $1 \le x \le 3$, $\frac{1}{x}$ is between 0 and π and here $\sin(\frac{1}{x}) > 0$. Thus $f'(x) > 0$ and so f is increasing. Thus L_7 underestimates, R_{13} overestimates

b) $f''(x) = \frac{1}{x^2}\cos(\frac{1}{x})(-\frac{1}{x^2}) - \frac{2}{x^3}\sin(\frac{1}{x}) = -(\frac{1}{x^4}\cos(\frac{1}{x}) + \frac{2}{x^3}\sin(\frac{1}{x}))$

for $1 \le x \le 3$, $\cos(\frac{1}{x})$ is also positive, so $f''(x) < 0$. Thus the fn is concave down and this is the reason T_{11} underestimates and M_7 overestimates.

6.1.20 a) Since f is increasing, L_{10} underestimates both I and R_{10}. Thus the ineq $|I - L_{10}| \le R_{10} - L_{10}$ can be written more simply as
$I - L_{10} \le R_{10} - L_{10}$. This is true since R_{10} overestimates I.

b) $T_{10} = \frac{R_{10} + L_{10}}{2} = 9.495$

c) $I - T_{10} = I - L_{10} - \frac{R_{10} - L_{10}}{2}$.

But $I - L_{10} \ge 0$. Thus $I - T_{10} \ge -\frac{R_{10} - L_{10}}{2}$

On the other hand, $I - L_{10} \le R_{10} - L_{10}$, so

$$I - T_{10} \le R_{10} - L_{10} - \frac{R_{10} - L_{10}}{2} = \frac{R_{10} - L_{10}}{2}$$

Combining, we get $|I - T_{10}| \le \frac{R_{10} - L_{10}}{2}$.

6.1.61 Since $f'(x) < 0$, f is decreasing. Therefore $R_n \le I \le L_n$. Also the graph shows that $f''(x) < 0$ (ie f' is decreasing), so f is concave down. Hence $T_n \le I \le M_n$. Since $T_n = \frac{L_n + R_n}{2}$, an average, then T_n must be larger than the smaller of the two numbers, here R_n. Thus $R_n \le T_n$. A graph shows that $M_n \le L_n$ for any decreasing function. Thus

$$R_n \le T_n \le I \le M_n \le L_n$$