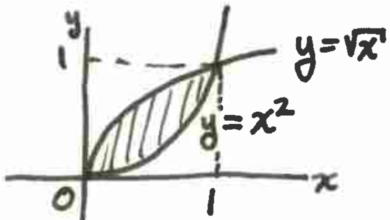
Assignment 3

Sec 7.1: 18, 20, 30, 46, 50

Sec 7.2: 10, 14, 18, 22, 43, 44, 48

7.1.18



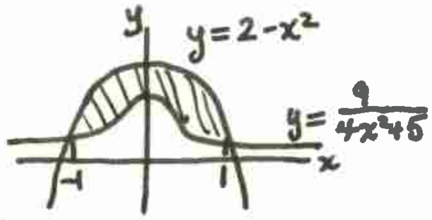
Intersections: $\sqrt{x}=x^2$

$\Rightarrow x = x^4$

$x=0$ or $x^3=1$, ie $x=1$

$$A = \int_0^1 (\sqrt{x}-x^2)\,dx = \tfrac{2}{3}x^{3/2}\Big]_0^1 - \frac{x^3}{3}\Big]_0^1 = \frac{2}{3}-\frac{1}{3}=\frac{1}{3}$$

7.1.20



Intersections: $2-x^2 = \frac{9}{4x^2+5}$

$\Rightarrow 8x^2+10-4x^4-5x^2 = 9$

$4x^4-3x^2-1 = 0$

$(4x^2+1)(x^2-1)=0$

$\Rightarrow \quad x=\pm 1$

$$A = 2\int_0^1 \left(2-x^2-\frac{9}{4x^2+5}\right)dx$$

$$= 2\left(2x-\frac{x^3}{3}\right)\Big]_0^1 - \frac{18}{5}\int_0^1 \frac{dx}{\left(\frac{2}{\sqrt5}x\right)^2+1}\cdot\frac{2}{\sqrt5}\cdot\frac{\sqrt5}{2}$$

$$= 2\left(2-\tfrac{1}{3}\right) - \frac{9}{\sqrt5}\arctan\left(\frac{2}{\sqrt5}x\right)\Big]_0^1$$

$$= \frac{10}{3} - \frac{9}{\sqrt5}\arctan\left(\frac{2}{\sqrt5}\right)$$

7.1.30 The figure is as in 7.1.18 though now are curves are $x=y^2$ and $x=\sqrt{y}$. Intersections are the same, so

$$A = \int_0^1 (\sqrt{y}-y^2)\,dy = \cdots = \frac{1}{3}$$

7.1.46 I have written to Paul Zorn about this question. Let's wait and see what he says. Doesn't look solvable to me!

7.1.50 $f'(x) = x^3 - \frac{1}{4x^3}$, so $1+(f'(x))^2 = 1+\left(x^3-\frac{1}{4x^3}\right)^2$

$$= 1 + x^6 - \frac{1}{2} + \frac{1}{16x^6}$$

$$= x^6 + \frac{1}{2} + \frac{1}{16x^6} = \left(x^3+\frac{1}{4x^3}\right)^2.$$