Solutions to Assignment 5

2.5.10 $f'(x) = 3x$, $f(0) = 37$

$f(x) = \frac{3}{2}x^2 + C$ so $37 = f(0) = C$

$\therefore f(x) = \frac{3}{2}x^2 + 37$ i.e. one solution

2.5.22 $y = \frac{1}{2}t^4 + \frac{3}{2}t^2$, $y' - \frac{2y}{t} = t^3$

Substitute y to test:

$\left(\frac{1}{2}t^4 + \frac{3}{2}t^2\right)' - \frac{2}{t}\left(\frac{1}{2}t^4 + \frac{3}{2}t^2\right) \stackrel{?}{=} t^3$

$2t^3 + 3t - t^3 - 3t \stackrel{?}{=} t^3$

$t^3 \stackrel{?}{=} t^3$. Yes it is satisfied for all t

2.5.26 $y = x+1$, $xy' + (y')^2 - y = 0$

Substitute:

$x((x+1)') + ((x+1)')^2 - (x+1) \stackrel{?}{=} 0 \quad \forall x$

$x \cdot 1 + (1)^2 - x - 1 \stackrel{?}{=} 0$

$0 \stackrel{?}{=} 0$ Yes, it is satisfied for all x

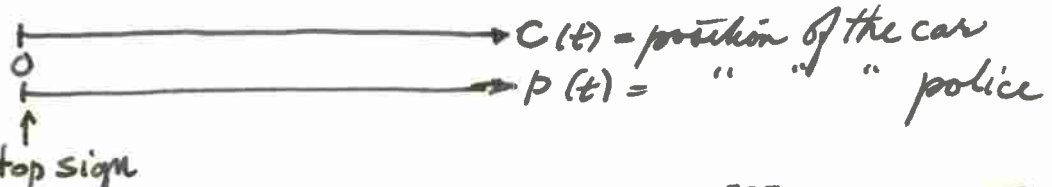
2.5.38 a) $v(t) = \frac{100}{(t+C)} \Rightarrow v' = -\frac{100}{(t+C)^2} = -\frac{1}{100}\frac{100^2}{(t+C)^2} = -\frac{1}{100}v$. Yes!

b) $v(0) = 5 \Leftrightarrow 5 = \frac{100}{C}$, so $C = 20$. Thus

$v(t) = \frac{100}{t+20}$. Thus $v(30) = \frac{100}{50} = 2$, $v(80) = \frac{100}{100} = 1$

This is an air resistance problem and the object clearly slowing down

2.5.46



The car is traveling at $80\text{ mph} = 80\frac{5280}{3600}\text{ ft sec}^{-1} = \frac{352}{3}\text{ ft sec}^{-1}$

If we take $t=0$ to be the time when the police car begins moving, Then $p(0) = 0$ and $C(0) = \frac{352}{3} \times 3\text{ sec} = 352$.