Solutions to Assignment 6

7.4.18 $\quad y' = 2e^{-y}, \; y(0) = 1$

$\frac{dy}{dx} = 2e^{-y}$, so separating gives $\quad e^{y}dy = 2dx$

Thus $\quad e^{y} = 2x + C$. At $(0,1)$ we have $e = 0 + C$, so $C = e$.

Therefore $e^{y} = 2x + e$ and so $\quad y = \ln(2x + e)$.

7.4.20 $\quad y' = \frac{x}{1+y^2}, \; y(2) = 3$

$\frac{dy}{dx} = \frac{x}{1+y^2}$, so separating gives $\quad (1+y^2)\,dy = x\,dx$

Therefore $\quad y + \frac{1}{3}y^3 = \frac{1}{2}x^2 + C$. At $(2,3)$ we get $\quad 3 + \frac{1}{3}3^3 = \frac{1}{2}2^2 + C$ or $C = 10$

Thus $\quad y + \frac{1}{3}y^3 = \frac{1}{2}x^2 + 10$. We cannot easily solve this for y, so we leave it as is and say that it <u>defines the solution y implicitly</u>

7.4.22 $\quad y' = x/y \Leftrightarrow \frac{dy}{dx} = \frac{x}{y} \Leftrightarrow \; y\,dy = x\,dx \Leftrightarrow \frac{1}{2}y^2 = \frac{1}{2}x^2 + C$

We want $\; y(2) = 3$, so $\frac{1}{2}3^2 = \frac{1}{2}2^2 + C$ or $C = \frac{9}{2} - \frac{4}{2} = \frac{5}{2}$. Hence $\frac{1}{2}y^2 = \frac{1}{2}x^2 + \frac{5}{2}$, or $y = \pm\sqrt{x^2+5}$. There appear to be two sol'ns here, but only one of them satisfies $y(2) = 3$. Which one is it?

7.4.26 $\quad y' = \sqrt{1-y^2} \Leftrightarrow \frac{dy}{dx} = \sqrt{1-y^2} \Leftrightarrow \frac{dy}{\sqrt{1-y^2}} = dx \Leftrightarrow \arcsin y = x$

So take $\quad y = \sin x$. Check it! Note that I didn't include a const because we just want one sol'n, not all solutions!

7.4.28 $\quad y' = 1 - y \Leftrightarrow \frac{dy}{dx} = 1 - y \Leftrightarrow \frac{dy}{1-y} = dx \Leftrightarrow -\ln|1-y| = x$

$$\Leftrightarrow |1-y| = e^{-x}$$

Thus $\quad 1 - y = \pm e^{-x}$. Since we just want one sol'n, pick a sign, say $+$.

Therefore $\; y = 1 - e^{-x}$. Check it!

7.4.30 $\quad y' = y\ln|y| \Leftrightarrow \frac{dy}{dx} = y\ln|y| \Leftrightarrow \frac{dy}{y\ln|y|} = dx \Leftrightarrow \int\frac{dy}{y\ln|y|} = x$.

Integrate this using the sub $u = \ln|y|$ to get $\ln(\ln|y|) = x$

Thus $\ln|y| = e^{x}$, $|y| = e^{e^{x}}$ and so $\quad y = e^{e^{x}}$ (ie pick a sign)