

Solutions to Assignment 7

$$8.2.36 \quad I = \int \frac{4x^2 - 3x + 2}{x(2x-1)^2} dx$$

First we look for constants A, B, C such that

$$\frac{4x^2 - 3x + 2}{x(2x-1)^2} = \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{(2x-1)^2}$$

$$\text{ie } 4x^2 - 3x + 2 = A(2x-1)^2 + Bx(2x-1) + Cx$$

$$\text{At } x=0: \quad 2 = A$$

Let's put this back in and simplify:

$$4x^2 - 3x + 2 = 2(4x^2 - 4x + 1) + Bx(2x-1) + Cx$$

$$4x^2 - 8x^2 - 3x + 8x + 2 - 2 = x(B(2x-1) + C)$$

$$-4x^2 + 5x = x(B(2x-1) + C)$$

Both sides have a factor of x (we should expect this) so cancel:

$$-4x + 5 = B(2x-1) + C$$

$$\text{At } x = \frac{1}{2}: \quad 3 = C$$

Substitute and simplify again:

$$-4x + 5 - 3 = B(2x-1)$$

$$-2(2x-1) = B(2x-1)$$

$$\text{So } B = -2.$$

$$\begin{aligned} \text{Thus } I &= 2 \int \frac{dx}{x} - 2 \int \frac{dx}{2x-1} + 3 \int \frac{dx}{(2x-1)^2} \\ &= 2 \ln|x| - \ln|2x-1| - \frac{3}{2} \frac{1}{2x-1} + C \end{aligned}$$

$$8.2.38 \quad I = \int \frac{x^3}{x^2+1} dx$$

The rational function in the integral is improper, so reduce it:

$$\frac{x^3}{x^2+1} = \frac{x^3+x-x}{x^2+1} = \frac{x(x^2+1)-x}{x^2+1} = x - \frac{x}{x^2+1}$$

$$\text{Thus } I = \int x dx - \int \frac{x}{x^2+1} dx = \frac{x^2}{2} - \frac{1}{2} \ln(x^2+1) + C$$