Assignment 8 Solutions

10.1.28 Area $= \int_1^\infty \left(\frac{1}{x} - \frac{1}{x+1}\right) dx$

$$= \lim_{t\to\infty} \left(\ln x - \ln(x+1)\right)\Big]_1^t$$

$$= \lim_{t\to\infty} \left(\ln \frac{t}{t+1} - \ln \frac{1}{1+1}\right)$$

$$= \ln\left(\lim_{t\to\infty} \frac{t}{t+1}\right) - \ln \frac{1}{2}$$

$$= \ln 1 + \ln 2$$

$= \ln 2$. Yes the area is finite

10.1.32 Volume $= \int_0^1 \left(\frac{\pi}{y^2} - \pi\right) dy$ $\qquad y = \frac{1}{x}$, so $x = \frac{1}{y}$

$$= \pi \int_0^1 \frac{dy}{y^2} - \pi$$

improper at $y = 0$, so:

$$\int_0^1 \frac{dy}{y^2} = \lim_{t\to 0} \int_t^1 \frac{dy}{y^2} = \lim_{t\to 0} -\frac{1}{y}\Big]_t^1 = \lim_{t\to 0} \left(-1 + \frac{1}{t}\right) = \infty$$

Thus the volume is infinite.

10.1.34 $\int_1^\infty \frac{dx}{x(1+x)} = \int_1^\infty \underbrace{\left(\frac{1}{x} - \frac{1}{1+x}\right)}_{\text{by partial fractions method}} dx$

$$= \lim_{t\to\infty} \left(\ln x - \ln(1+x)\right)\Big]_1^t$$

$$= \lim_{t\to\infty} \ln \frac{x}{1+x}\Big]_1^t$$

$$= \lim_{t\to\infty} \left(\ln \frac{t}{1+t} - \ln \frac{1}{1+1}\right)$$

$$= \ln 1 - \ln \frac{1}{2} = 0 + \ln 2 = \ln 2.$$

10.1.38 $\underbrace{\int_1^3 \frac{dx}{(x-2)^{1/3}}}_{\text{improper at } x=2,} = \int_1^2 \frac{dx}{(x-2)^{1/3}} + \int_2^3 \frac{dx}{(x-2)^{1/3}}$

so both of these would need to converge.