Assignment 9 Solutions

9.1.20 $f(x) = \frac{1}{x^2}$, $x_0 = 1 \Rightarrow f(1) = 1$

$f'(x) = -\frac{2}{x^3}$, $f'(1) = -2!$

$f''(x) = \frac{3!}{x^4}$, $f''(1) = 3!$

$f'''(x) = -\frac{4!}{x^5}$, $f'''(1) = -4!$

$f^{IV}(x) = \frac{5!}{x^6}$, $f^{IV}(1) = 5!$

$f^{V}(x) = -\frac{6!}{x^7}$, $f^{V}(1) = -6!$

$f^{VI}(x) = \frac{7!}{x^8}$

So $P_5(x) = 1 - 2!(x-1) + \frac{3!}{2!}(x-1)^2 - \frac{4!}{3!}(x-1)^3 + \frac{5!}{4!}(x-1)^4 - \frac{6!}{5!}(x-1)^5$

$= 1 - 2(x-1) + 3(x-1)^2 - 4(x-1)^3 + 5(x-1)^4 - 6(x-1)^5$

A plot shows that $|f(x) - P_5(x)| \le .01$ for $.69 \le x \le 1.36$

9.1.22 $f(x) = \sqrt{x}$, $x_0 = 4 \Rightarrow f(4) = 2$

$f'(x) = \frac{1}{2}x^{-1/2}$, $f'(4) = \frac{1}{4}$

$f''(x) = -\frac{1}{4}x^{-3/2}$, $f''(4) = -\frac{1}{32}$

$f'''(x) = \frac{3}{8}x^{-5/2}$, $f'''(4) = \frac{3}{8 \times 2^5}$

$f^{IV}(x) = -\frac{15}{16}x^{-7/2}$, $f^{IV}(4) = -\frac{15}{16 \times 2^7}$

$f^{V}(x) = \frac{105}{32}x^{-9/2}$, $f^{V}(4) = \frac{105}{32 \times 2^8}$

So $P_5(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{32 \times 2!}(x-4)^2 + \frac{3}{8 \times 2^5 \times 3!}(x-4)^3$

$- \frac{15}{16 \times 2^7 \times 4!}(x-4)^4 + \frac{105}{32 \times 2^8 \times 5!}(x-4)^5$

$= 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3 - \frac{5}{16384}(x-4)^4$

$+ \frac{7}{131072}(x-4)^5$

A plot shows that $|f(x) - P_5(x)| \le .01$ when $1.2 \le x \le 7.5$